ACA 2009

Applications of Computer Algebra

15th International Conference

Applications of Computer Algebra

ACA 2009

École de technologie supérieure (ÉTS), June 25-28, 2009
Montréal, Québec, Canada

ROOM ASSIGNMENTS AT A GLANCE

The conference opening ceremony will be held in the Nortel auditorium (room A-1600) of Building A. All other activities of the conference will take place in building B.

Sessions		Rooms
1.	Computer Algebra in Education - Talks Computer Algebra in Education – Workshops	B-4408 B-2404
2.	Interaction Between Computer Algebra and Interval Computations	B-2624
3.	Applications and Libraries development in Derive	B-3432
4.	Elimination Theory and Applications	B-2620
5.	Chemistry and Computer Algebra	B-2624
6.	Applications of Math Software to Mathematical Research	B-2620
7.	Computer Algebra for Dynamical Systems and Celestial Mechanics	B-3432
8.	Analogy in Reasoning and Construction	B-3420
9.	Symbolic and Numeric Computation	B-3432
10.	Algebraic and Algorithmic Aspects of Differential and Integral Operators	B-2620
11.	High-Performance Computer Algebra	B-2624
12.	Nonstandard Applications of Computer Algebra – Thursday Nonstandard Applications of Computer Algebra – Friday	B-3432 B-3420
13.	Symbolic and numeric approaches to dynamical modeling and simulation	B-4404
14.	Algorithms for Parametric Systems and their Applications	B-3432

Conference Desk location:
Wednesday, June 24, Lobby of building B
Thursday, June 25, 7:45 to 10:45, near Nortel room A-1600
Thursday, June 25, after 10:45, room B-4418
Friday to Sunday, June 25-28, room B-4418.

Conference Desk opening hours:
Wednesday, June 24, from 17:00 to 19:00
Thursday, June 25, from 7:45 to 17:30
Friday, June 26, from 7:45 to 12:30
Saturday, June 27, from 8:00 am to 17:00
Sunday, June 28, 8:00 to 12:30

ÉTS opening hours:
Thursday and Friday: 6:30am to 11pm
Saturday and Sunday: 7:30am to 6pm

ACA 2009 Website: http://aca2009.etsmtl.ca

WELCOME TO ACA 2009

Dear colleagues,

It is our pleasure to welcome you to the 15th International Conference on Applications of Computer Algebra (ACA 2009).

As the local organizing committee, we wish to express our gratitude to the speakers, the program co-chairs and all the members of the program committees. We would also like to thank the following people for their contribution to the success of ACA 2009.

Danielle Bouthot, our webpage designer and computer wizard
Gilles Fontaine, who made our job easier by dealing with many local logistics problems
Martine Guertin, for taking care of some of the grunt work
Sylvain Huneault and Georges Tremblay, our wonderful multimedia people

Many thanks go out to our University's administration for their support. In particular, we would like to thank Claude Olivier, Director of Academic Affairs and Executive Director, for accepting our invitation to give the opening address.

Thanks to all those persons whose names do not appear here but who contributed to the organization of ACA 2009.

Finally, our gratitude goes out to our sponsors for their contributions: École de technologie supérieure (ÉTS), COOP ÉTS, MathWorks, International Association for Mathematics and Computers in Simulation (IMACS), Éditions du renouveau pédagogique (ERPI), Wiley, Pearson and Maplesoft.

Have a wonderful conference!

Michel Beaudin, Gilles Picard and Kathleen Pineau

Michel Beaudin, Kathleen Pineau and Gilles Picard
Photo by Michael Wester, ACA 2001
Near Socorro, New Mexico

BRIEF HISTORY OF THE ACA SERIES

The 1st International IMACS Conference on Applications of Computer Algebra was held in 1995 at the University of New Mexico in Albuquerque, New Mexico, USA. IMACS stands for "International Association for Mathematics and Computers in Simulation". Since 2001, the conference retains only the title "Applications of Computer Algebra". This conference is an annual meeting devoted to promoting the applications and development of computer algebra/symbolic computation. Topics include computer algebra/symbolic computation in engineering, the sciences, medicine, pure and applied mathematics, education, communication and computer science among many others.

INFORMATION AND GENERAL REMARKS

Identification badge

Your conference materials include your personal identification badge as well as badges for your guests. The delegate's badge is, in fact, a magnetic card. You need this card to have access to the computer labs (rooms B-2402, B-2404 and B-2406), the faculty lounge (B-4502), the meeting room (B-4404), and the ÉTS sports facilities (B-3009), locker rooms (B-3506 or B-3516), and showers. Your magnetic card is also required when getting coffee break refreshments and lunches.

Conference Desk

A Conference Desk will serve as the information centre of ACA 2009, where delegates can pick up their conference documents and get information.

Conference Desk location:	Wednesday, June 24, Lobby of building B Thursday, June 25, 7:45 to 10:45, near Nortel room A-1600 Thursday, June 25, after 10:45, room B-4418 Friday to Sunday, June 26-28, room B-4418.
Conference Desk opening hours:	Wednesday, June 24, from 17:00 to 19:00 Thursday, June 25, from 7:45 to 17:30 Friday, June 26, from 7:45 to 12:30 Saturday, June 27, from 8:00 am to 17:00 Sunday, June 28, 8:00 to 12:30

Information distribution

A Message Board is set up near the Conference Desk for dissemination of information of possible interest to delegates.

A person wishing to display information or leave a message can do so at the Conference Desk during the hours that the desk is open. These messages will be posted on the Message Board. However, staff at the desk will try to locate a participant in the event of an emergency.

Email services

Email and internet access is provided in computer labs B-2402, B-2404 and B-2406. You will need your magnetic card to have access to these rooms. For access to **computers in labs**:

compte d'usager (username): ACA2009
mot de passe (password): ACA2009
domaine (domain): ENS (by default)

For **wireless** access at ÉTS, connect to Network ETS-PUBLIC:

compte d'usager (username): wifi-SEG@etsmtl.ca
mot de passe (password): seg-E2009

Meeting rooms

Meeting room B-4404, when not occupied, as well as rooms B-4410, B-4416 and the faculty lounge, B-4502, can be used by delegates for impromptu meetings and discussions at any time during the conference.

Exhibits

All conference delegates are encouraged to visit the MathWorks booth. You will find it in the same room as the Conference Desk, B-4418.

Photocopies

Inquire at the Conference Desk about photocopying possibilities.

Sports Facilities

Your magnetic card will give you access to ÉTS's Sports Facilities (B-3009), locker rooms, and showers (B-3506 or B-3516). If you choose to use the facilities, bring a lock and a towel. The sports facilities are indicated by “Service des sports” on the building B third floor plan of the Maps section.

Presentation support

Each conference room has a computer (Pentium class, Windows XP), a projector (1024x768), Internet access, and all the necessary laptop to projector connections.

Technical support will be more easily accessible on Thursday and Friday than on Saturday and Sunday. Speakers are therefore encouraged to test their material (rooms B-4410, B-4416 can be used) before the weekend.

ViewScreens for Voyage 200 and TI Nspire, as well as a traditional transparency projector are also available.

The following software is available on all computers: Derive 6.1, Maple 12, Matlab 2008, Mathcad 2001 Pro, DPGraph, Microsoft Office 2007 (French version), Statgraphics Centurion, Mozilla Firefox 3, Voyage 200 emulator and Acrobat reader 9.

For any information concerning technical questions, contact Gilles Picard by phone (see below) or by email at gilles.picard@etsmtl.ca.

Contact information

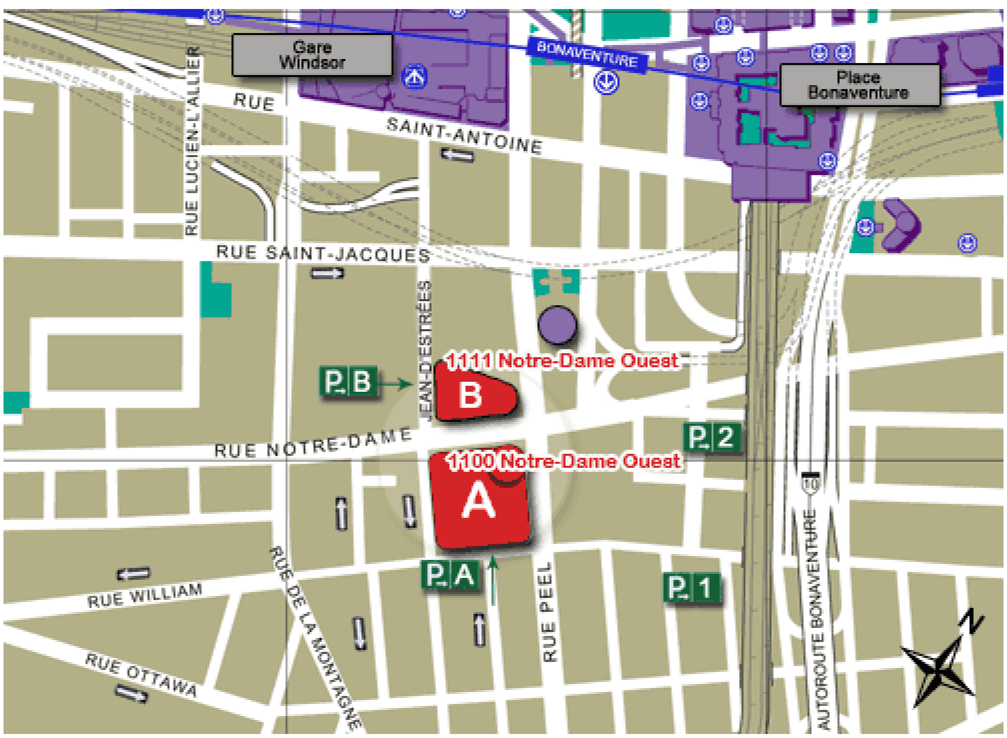
In the event you need more information about ACA 2009 than the Conference Desk staff is able to provide, contact the following persons:

for general information	Michel Beaudin	office room number B-2532 tel.: 514-396-8511 (office) 514-932-7098 (home)
	Kathleen Pineau	office room number B-2556 tel.: 514-396-8614 (office)
for technical support	Gilles Picard	office room number B-2530 tel. : 514-396-8500 (office) 514-833-7447 (cell)

Maps

Area Map of venue

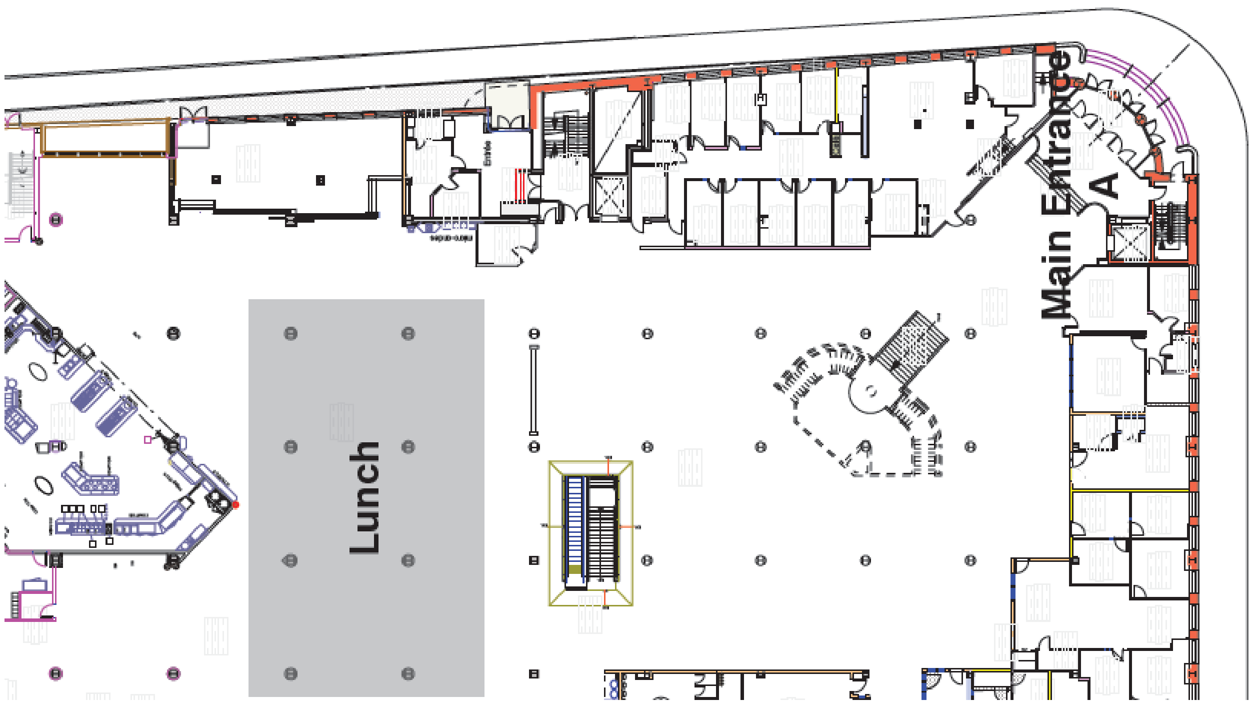
The ACA 2009 conference will be held at École de technologie supérieure (ÉTS) situated at the corner of Notre-Dame and Peel streets in downtown Montreal.



Building A, ground floor and first floor

Lunch on Thursday and Saturday will take place in a reserved area of the University cafeteria situated on the ground floor of building A, 1100 Notre-Dame street West.

← Notre-Dame street →



The conference opening ceremony will be held in the Nortel auditorium (A-1600) situated on the first floor of building A.

← Notre-Dame street →

Building B, ground floor and second floor

The Welcome Reception will be held in the lobby and Pub of building B, 1111 Notre-Dame street West.

← Notre-Dame street →

Computer labs and two conference rooms are situated on the second floor of building B.

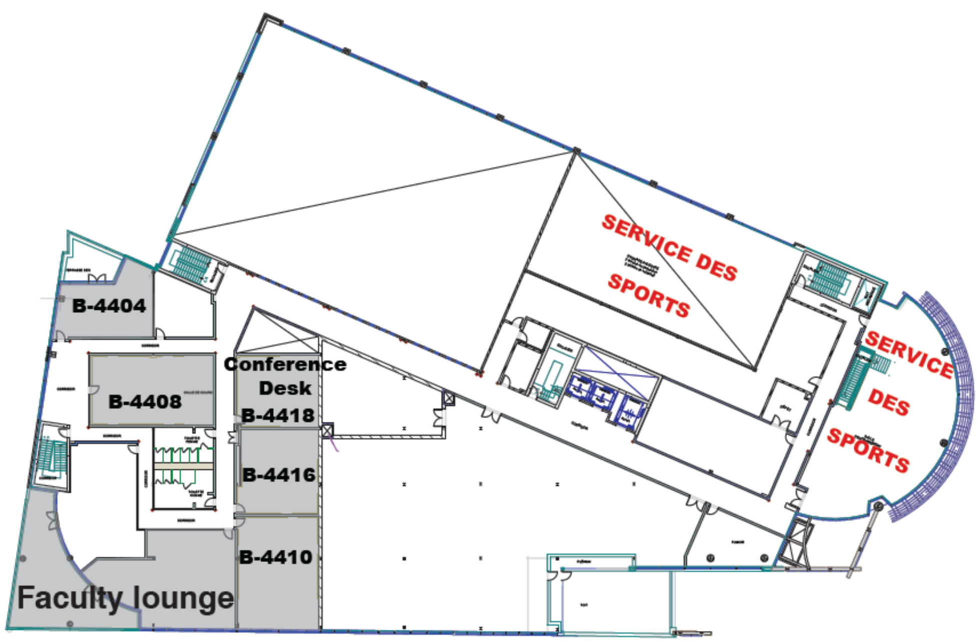
← Notre-Dame street →

Building B, third and fourth floors

The sports facilities (Service des sports) and two conference rooms are situated on the third floor of building B.

←Notre-Dame street →

The Conference desk, Faculty lounge and many conference rooms are situated on the fourth floor of building B.



←Notre-Dame street →

SOCIAL EVENTS

Pre-registration and Welcome Reception

A Welcome Reception will be held Wednesday, June 24th, from 17:00 to 19:00. The conference desk will be set up in the lobby of building B (1111 Notre-Dame street West) where delegates can pick up their conference documents and get information. Delegates will then be invited to join their colleagues at the University Pub for cocktails.

Optional Friday Excursion

Two activities in Montréal are offered for Friday, June 26th, 2009. When you registered for ACA 2009, you indicated which excursion you wanted to take, if any, and your excursion fee was added to your registration fee. You will find the ticket for your choice of excursion in your conference pack. Your excursion ticket is required as proof of registration to the outing.

There are a few places still available to join one of the walking tours. Information on possibilities to join a tour is available at the Conference Desk.

Lunch and walking tour of Chinatown (35$) Contact: Kathleen Pineau
We meet at 12:35 in front of building B

We leave ÉTS on foot at 12:45 to explore Chinatown with a professional guide. We meet in front of building B, 1111, Notre-Dame St. West, at 12:35. We regret that we cannot wait for latecomers. Latecomers will not be reimbursed.

This tour starts with lunch where you will discover Chinese knowhow while tasting traditional dishes and Dimsum. The guide will then take you on a walking tour of the neighbourhood. Discover the history and specificities of this ethnic group that has contributed to the cultural heritage of this country. You will visit places of cultural significance: Buddhist temple for devotions, worship store for objects associated with chance and ancestry, herbalist, grocery store, etc. Chinese religious rituals, family and social traditions will be the overarching themes of this tour.

Lunch is included in the cost of this tour. Vegetarian menus or menus for those with food allergies are available. You will have indicated any special needs of this sort when registering. Nonetheless, please bring this to the attention of the tour guide.

The tour ends approximately at 16:30 pm.

Walking tour of Historical Old Montréal (15$) Contacts: Michel Beaudin and Gilles Picard
We meet at 14:20 in front of building B

We leave ÉTS on foot at 14:30 pm to explore Old Montréal with a professional guide. We meet in front of building B, 1111, Notre-Dame St. West, at 14:20. We regret that we cannot wait for latecomers. Latecomers will not be reimbursed.

From the founding of Ville-Marie to today's metropolitan reality, discover the important historical events and transformations of Montréal's oldest borough. Learn some of the secrets of Montréal's historical neighbourhood which is rich in history, architecture and monuments.

This tour concludes approximately at 16:30 pm near the Notre-Dame Basilica which you can choose to visit on your own.

Walking tours will take place rain or shine. Wear comfortable walking shoes and a smile.

Banquet dinner

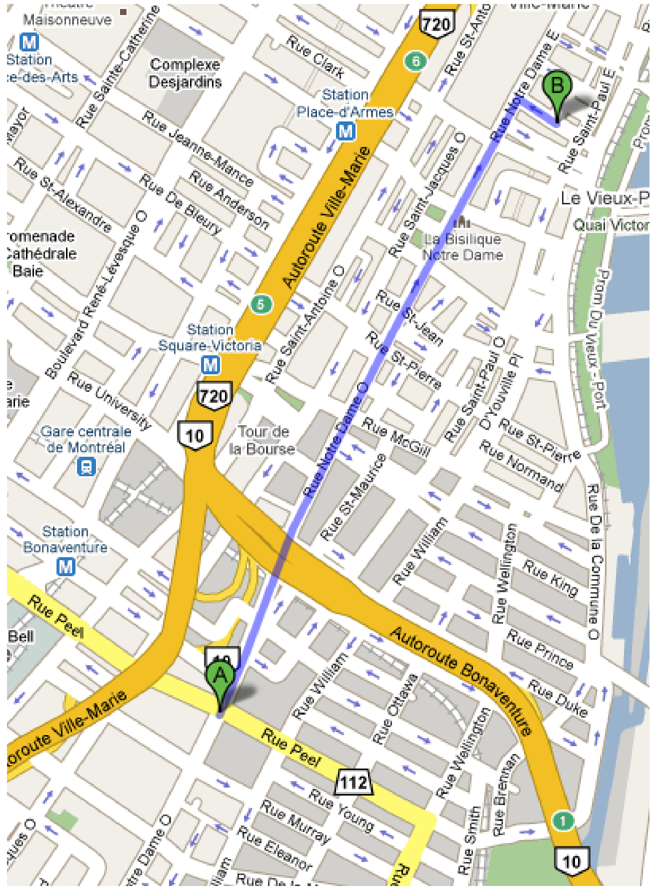
The banquet dinner will be held on Saturday, June 27th, 2009 at the restaurant, **Auberge Saint-Gabriel**, in Old Montréal. The restaurant is a 15 to 20-minute walk from ÉTS and is situated at 426 Saint-Gabriel street.

The cocktail preceding the banquet dinner will start at 18:30 pm. Cocktail and dinner will take place in the Grenier room of the Auberge.

Sorry, the banquet room is not wheel chair accessible.

Vegetarian menus or menus for those with food allergies have been planned for those who have indicated such needs when registering. Simply make yourself known to the staff. Be aware that special menus are available to you IF AND ONLY IF YOU mentioned it when registering for ACA 2009.

A: École de technologie supérieure, venue.
B: Auberge Saint-Gabriel, 426 Saint-Gabriel.



Lunch on Thursday and Saturday

Registration fees cover lunch on Thursday and Saturday.

A coupon for lunch on Thursday is in your conference pack. You must hand it in to the cashier at the cafeteria in order to pay for lunch.

As the cafeteria is closed on Saturday, a buffet lunch will be provided. You will need your conference badge in order to be served.

For any questions you may have concerning the social aspects of ACA 2009, inquire at the Conference Desk or contact Kathleen Pineau at (514) 396-8614 or kathleen.pineau@etsmtl.ca.

CONFERENCE OVERVIEW

This section contains an overview schedule of the ACA 2009 conference. More information on sessions and abstracts are available in the following sections of this document.

Thursday, June 25, 2009

8:00-9:15	**Registration/Coffee** (near Room A-1600)				
9:15 – …	Welcome – Local organizers (room A-1600) MICHEL BEAUDIN, GILLES PICARD AND KATHLEEN PINEAU				
… – …	Welcome address – CLAUDE OLIVIER Director of Academic Affairs and Executive Director École de technologie supérieure				
… – 10:00	Welcome – ACA conference *constants* STANLY STEINBERG AND MICHAEL WESTER Information – Organisation KATHLEEN PINEAU				
10:00-10:30	**Coffee Break** (near room A-1600) Delegates then move to building B				
Let the sessions begin!	**Room B-4408**	**Room B-2620**	**Room B-2624**	**Room B-3432**	**Room B-4404**
	Session 1 Computer Algebra in Education	**Session 10** Algebraic and Algorithmic Aspects of Differential and Integral Operators	**Session 2** Interaction Between Computer Algebra and Interval Computation	**Session 14** Session on Algorithms for Parametric Systems and their Applications	**Session 13** Symbolic and Numeric Approaches to Dynamical Modeling and Simulation
10:30-11:00	Beaudin, Picard	Barkatou	Makino	Suzuki	Gerhard
11:00-11:30	Etchecopar, Nadal, Villeneuve	Cluzeau	Berz	Schost, Dahan, Kadri	Moreno Maza
11:30-12:00	Buteau	Guo, Sit, Zhang	Neher	Coral, Gonzalez-Vega	Reid
12:00-12:30	Char, Johnson, Augenblick	Koutschan	Wittig	Moroz	Mani
12:30-14:00	**Lunch** (coupon) Room: reserved section in the cafeteria, building A				

Thursday, June 25, 2009 - after lunch

	Room B-4408	**Room B-2620**	**Room B-2624**	**Room B-3432**	**Room B-4404**
	Session 1 Computer Algebra in Education	**Session 10** Algebraic and Algorithmic Aspects of Differential and Integral Operators	**Session 2** Interaction Between Computer Algebra and Interval Computation	**Session 14** Session on Algorithms for Parametric Systems and their Applications	**Session 13** Symbolic and Numeric Approaches to Dynamical Modeling and Simulation
14:00-14:30	P. Leinbach C. Leinbach	Labahn	Kraemer	Chen, Lemaire, Moreno Maza, Xia, Xiao, Xie	Wittkopf, A. Roche
14:30-15:00	C. Leinbach P. Leinbach	Liu, Reid	Van Deun	Chen, Davenport, May, Moreno Maza, Xia, Xiao, Xie	Tidefelt
15:00-15:30	Monagan, Ogilvie	Pritchard, Sit	Auer	Verschelde, Piret	Wolf
15:30-16:00	**Coffee Break** Faculty Lounge, room B-4502				
	Session 1 Computer Algebra in Education	**Session 10** Algebraic and Algorithmic Aspects of Differential and Integral Operators	**Session 11** High-Performance Computer Algebra	**Session 12** Nonstandard Applications of Computer Algebra	**Session 13** Symbolic and Numeric Approaches to Dynamical Modeling and Simulation
16:00:16:30	Curts	Quadrat	Davenport	Roanes-Lozano, Hernando, Alonso	Spivey, Hedengren, Edgar
16:30-17:00	A.Mylläri, T.Mylläri	Regensburger, Rosenkranz, Middeke	Lacelle, Schost	Wester, Yaacob, Steinberg	Postma, Shmoylova
17:00-17:30		Shemyakova, Mansfield	Robinson, Müller Cooperman, Kunkle,	Aguilera, J. L. Galán, Padilla, Rodríguez	Wolf
17:30-18:00			Verschelde	Recio, Sendra, Tabera, Villarino	Bardhan

Friday, June 26, 2009

8:00-8:30	**Coffee** (Faculty Lounge, room B-4502)				
	Room B-4408	**Room B-2620**	**Room B-2624**	**Room B-3432**	**Room B-3420**
	Session 1 Computer Algebra in Education	**Session 10** Algebraic and Algorithmic Aspects of Differential and Integral Operators	**Session 11** High-Performance Computer Algebra	**Session 3** Applications and Libraries Development in Derive	
8:30-9:00	8:45-9:30 Saint-Aubin (Keynote)	Baran	D. Roche	Böhm	
9:00-9:30		Umemura	Ding, Schost, Moreno Maza	Roanes-Lozano, González	
9:30-10:00	Buteau, Marshall, Jarvis, Lavicza	Zhang	Xie, Moreno Maza	Schmidt	
10:00-10:30	**Coffee Break** (Faculty Lounge, B-4502)				
	Session 1 Computer Algebra in Education	**Session 6** Application of Math Software to Mathematical Research	**Session 11** High-Performance Computer Algebra	**Session 3** Applications and Libraries Development in Derive	**Session 12** Nonstandard Applications of Computer Algebra
10:30-11:00	Ellis, Bauldry	Benghorbal	Johnson, Meng	A.García, F.García, Rodríguez, De la Villa	Garcia
11:00-11:30	Bokhari, Yushau	Garcia-Puente	Dumas, Saunders, Youse	Beaudin	Hernández, García, Rivas, Marco, E. Sáenz-de-Cabezón, Pérez-Moreno, F. J. Sáenz-de-Cabezón
11:30:12:00	Collins	Koutschan	Pan, Li, Moreno Maza	Aguilera, J. L. Galán, M.Á Galán, Padilla, Rodríguez	Bauldry
12:00-12:30	Lopez	Lebl	Johnson, Krandick, Richardson, Ruslanov		Hernando
12:30- …	**Optional excursion** (see excursion section for more information)				

Saturday, June 27, 2009

8:00-8:30	**Coffee** (Faculty Lounge, B-4502)				
	Room B-4408	**Room B-2620**	**Room B-2624**	**Room B-3432**	**Room B-3420**
	Session 1 Computer Algebra in Education	**Session 6** Application of Math Software to Mathematical Research	**Session 5** Chemistry and Computer Algebra	**Session 9** Symbolic and Numeric Computation	**Session 8** Analogy in Reasoning and Construction
8:30-9:00		Lichtblau	Whitehead	Kai	8:30 Opening 8:35-9:10 Barnett 9:15-10:00 Bouchard
9:00-9:30	Böhm	McGrail	Lewis, Coutsias	Stefanescu	
9:30-10:00	Aguilera, Fernández J. L. Galán, Mérida, Mora, Rodríguez	Medina	Yeh	Murakam	
10:00-10:30	**Coffee Break** (Faculty Lounge, B-4502)				
10:30-11:00	Yaacob, Wester, Steinberg	A. Mylläri	Nie	Janovitz Freireich	10:30-11:15 Toussaint 11:20-12:05 Griffiths 12:10-12:30 General discussion
11:00-11:30	Jeffrey	T. Mylläri	Ogilvie	Poteaux	
11:30-12:00	Davenport	Verschelde		Bates	
12:00-12:30	Doke	Kaltofen	Scarlete, Heverly-Coulson, Dostie, Gagnon		
12:30-14:00	**Lunch** (buffet) Room: reserved section in the cafeteria, building A				
	Session 1 Computer Algebra in Education	**Session 4** Elimination Theory and Applications	**Session 5** Chemistry and Computer Algebra	**Session 9** Symbolic and Numeric Computation	**Session 8** Analogy in Reasoning and Construction
14:00-14:30	Böhm WIRIS Workshop Room B-2404	Yang, Zeng, Zhang	Henderson, Yildirim	Guan	14:00-14:45 Thagard 14:50-15:30 Further discussion
14:30-15:00		Lewis	Fraser	Moroz	
15:00-15:30		Minimair, Kapur			
15:30-16:00	**Coffee Break** (Faculty Lounge, B-4502)				
16:00-17:00	Jarvis GeoGebra Workshop Room B-2404	**ACA-WG meeting** (room B-4404)			
18:30-....	**Cocktail and banquet** Le Saint-Gabriel restaurant in Old Montréal				

Sunday, June 28, 2009

8:00-8:30	**Coffee** (Faculty Lounge, B-4502)		
	Room B-4408	**Room B-2620**	**Room B-3432**
	Session 1 Computer Algebra in Education	**Session 4** Elimination Theory and Applications	**Session 7** Computer Algebra for Dynamical Systems and Celestial Mechanics
8:30-9:00		Chen, Moreno Maza, Xia, Yang	Bruno, Edneral
9:00-9:30	Schmidt	Bates	Markovski, Chuluunbaatar, Gusev, Vinitsky
9:30-10:00	Tanguay, Boileau	Palancz, Zaletnyik, Awange, Lewis	A. Mylläri, T. Mylläri, Rostovtsev, Vinitsky
10:00-10:30	**Coffee Break** (Faculty Lounge, B-4502)		
10:30-11:00	10:30-11:15 Caron (Keynote) 11:15-12:30 Caron, Jarvis, Pineau (task discussion)	Bard	Vassiliev
11:00-11:30		Lundqvist	
11:30-12:00		Li, Moreno Maza, Pan	
12:00-12:30			
12:35-…	**Room B-4408** Closing ceremony STANLY STEINBERG AND MICHAEL WESTER Information on the next ACA meeting		

SESSION SCHEDULES AND ABSTRACTS

The following pages contain schedules and abstracts for all 14 sessions. Sessions are listed in the same order as they are on the ACA 2009 Website. Session abstracts follow session schedules and each group of abstracts is listed in alphabetical order by main presenter. For more information, please see the ACA 2009 website http://aca2009.etsmtl.ca.

1. Computer Algebra in Education

org : Kathleen Pineau, École de technologie supérieure, Canada (chair)
Michael Wester, University of New Mexico, USA (chair)

Alkis Akritas, University of Thessaly, Greece
France Caron, Université de Montréal, Canada
Daniel H. Jarvis, Nipissing University, Canada
Bernhard Kutzler, Austrian Center for Didactics of Computer Algebra (ACDCA), Austria
Bill Pletsch, Central New Mexico Community College (CNM), USA

Room: B-4408 for talks
B-2404 for workshops

Schedule

Thursday June 25th

10:30 – 11:00	MICHEL BEAUDIN AND GILLES PICARD Ten Years of Using Symbolic TI Calculators at ETS
11:00 – 11:30	PHILIPPE ETCHECOPAR, JORDI NADAL AND JEAN-PHILIPPE VILLENEUVE Integrating CA in Modelling-Simulation Approaches in the Mathematics Courses of the Science Programs at the CEGEP of Rimouski
11:30 – 12:00	CHANTAL BUTEAU A Sustained Integration of a Computer Algebra System in University Mathematics Education at Brock University
12:00 – 12:30	BRUCE CHAR, JEREMY JOHNSON AND DAVID AUGENBLICK Using Maple and Maple TA in a course about technical computing
14:00 – 14:30	PATRICIA LEINBACH AND CARL LEINBACH Using Forensic Investigations and CAS to Motivate Student Interest in Mathematics I
14:30 – 15:00	PATRICIA LEINBACH AND CARL LEINBACH Using Forensic Investigations and CAS to Motivate Student Interest in Mathematics II
15:00 – 15:30	M. B. MONAGAN AND J. F. OGILVIE Teaching and learning mathematics with symbolic computation
16:00 – 16:30	JAIME CURTS Teaching Principal Component Analysis in Minitab®
16:30 – 17:00	ALEKSANDR MYLLÄRI AND TATIANA MYLLÄRI CAS in Teaching Basics of Statistical Learning

Friday June 26th

8:45 – 9:30	YVAN SAINT-AUBIN – KEYNOTE SPEAKER Animations in Mathematics
9:30 – 10:00	CHANTAL BUTEAU, NEIL MARSHALL, DANIEL JARVIS AND ZSOLT LAVICZA Integrating Computer Algebra Systems in Post-Secondary Mathematics Education: Preliminary Results of a Literature Review
10:30 – 11:00	WADE ELLIS AND WILLIAM C. BAULDRY Using and Creating A-C-R Documents for Mathematics Instruction with Computer Algebra
11:00 – 11:30	M. A. BOKHARI AND B. YUSHAU Application of CAS to the classical definition of limit of function
11:30 – 12:00	KEN COLLINS Analysing Power Series using Computer Algebra and Precalculus Techniques
12:00 – 12:30	ROBERT LOPEZ Resequencing of Skills and a CAS You Don't Have to Teach

Saturday June 27th

9:00 – 9:30	JOSEF BÖHM Which CAS can fill the gap?
9:30 – 10:00	GABRIEL AGUILERA, ÁLVARO FERNÁNDEZ, JOSÉ LUIS GALÁN, ENRIQUE MÉRIDA, ÁNGEL MORA AND PEDRO RODRÍGUEZ Scilab and Maxima Environment: Towards free software in Numerical Analysis
10:30 – 11:00	YUZITA YAACOB, MICHAEL J. WESTER AND STANLY STEINBERG Developing an Automated Learning Assistant for Vector Calculus
11:00 – 11:30	DAVID J. JEFFREY Getting from x to y without crashing
11:30 – 12:00	JAMES DAVENPORT A Comparison of Equality in Computer Algebra and Correctness in Mathematical Pedagogy
12:00 – 12:30	JIRO DOKE Using Symbolic Math in Engineering Education
14:00 – 15:30	JOSEF BÖHM WIRIS Exploration (Workshop in computer lab B-2404)
16:00 – 17:00	DANIEL JARVIS *GeoGebra Exploration* (Workshop in computer lab B-2404)

Sunday June 28th

9:00 – 9:30	KARSTEN SCHMIDT Mathematics Education with a Handheld CAS – The Students' Perspective
9:30 – 10:00	DENIS TANGUAY AND ANDRÉ BOILEAU Using CAS in Letter-Symbolic Algebra at the Secondary Level : a Classroom Activity
10:30 – 11:15	FRANCE CARON – KEYNOTE SPEAKER Technology in Mathematics Education: From Meaning to Purpose
11:15 – 12:30	FRANCE CARON, DANIEL JARVIS AND KATHLEEN PINEAU Finale - Discussion “What constitutes a good CAS task?”

Abstracts

GABRIEL AGUILERA, ÁLVARO FERNÁNDEZ, JOSÉ LUIS GALÁN, ENRIQUE MÉRIDA, ÁNGEL MORA AND PEDRO RODRÍGUEZ, University of Málaga, Spain

[Saturday, June 27th, 9:30, B-4408]

Scilab and Maxima Environment: Towards free software in Numerical Analysis

One of the main objectives of this work is to present the possibilities that free mathematical software can provide for teaching Mathematics in University. Proprietary software holds several problems for students and teachers. Licenses for students are, in many cases, too expensive. For teachers in our University, the license price is not the only problem: we have to share a software which is centralized in a server for all the University. This server can be saturated and we even have had problems when doing an official exam.

In subjects such as Numerical Analysis, the most common software used is Matlab which is an expensive proprietary software. In our work, we first analyse the different alternatives to this Matlab within free software and later we explain the reasons for choosing Scilab + Maxima.

For the exercises we develop in the computer lab, Scilab presents a similar calculus engine and, Maxima supplies the lack of symbolic calculation of Scilab.

On the other hand, the main issue for us in using Scilab is its interface. Matlab interface is really good, and both teachers and students would miss this proprietary software for this reason.

In this talk we will present the environment **Scilab UMA** developed by us as an alternative to Matlab. This environment connects Scilab (for numerical analysis) and Maxima (for symbolic computations). Furthermore, the developed interface is in our opinion, at least, as powerful as the interface of Matlab.

Finally, as we have adapted all the material we use in computer lectures, we will use Scilab UMA next year in the Numerical Analysis subjects we teach.

MICHEL BEAUDIN AND GILLES PICARD, École de technologie supérieure, Canada
[Thursday, June 25th, 10:30, B-4408]
Ten Years of Using Symbolic TI Calculators at ETS

Since September 1999, the TI-92 Plus or TI-89 symbolic calculator has been a compulsory purchase for new students entering our engineering school and the Voyage 200 is now being used since September 2002. Looking back at these ten years of working with a computer algebra system on every student's desk, one could ask: did the introduction of this hand-held technology really forced teachers to re-assess their goals in teaching mathematics? For some teachers – in fact, less than we could have expected−, the answer is "yes". But what really has changed? Some exam questions have changed because students at ETS have access to their calculator when writing tests, almost all the time. But the curriculum did not really change: many professors still continue to ignore the power of computer algebra as if computer algebra serves only as a substitute for pencil and paper techniques or only to illustrate concepts. Many are reluctant to really integrate CAS as a working tool for students. Different parts of the curriculum or different mathematics courses are still connected in a very poor manner. The talk will give examples of what a daily use of computer algebra in the classroom should have produced: a better appropriation of (many) mathematical concepts. Perhaps mathematics teachers should start to question their role. Computer algebra systems are here to stay. Not using them won't make them disappear!

JOSEF BÖHM, Austrian Center for Didactics of Computer Algebra (ACDCA), Austria
[Saturday, June 27th, 14:00, computer lab B-2404]
WIRIS Exploration (Workshop)

A workshop will be offered for those participants interested in exploring the WIRIS software. See the WIRIS website (http://www.wiris.com/index.php?lang=en) for more information on the software.

JOSEF BÖHM, Austrian Center for Didactics of Computer Algebra (ACDCA), Austria
[Saturday, June 27th, 9:00, B-4408]
Which CAS can fill the gap?

As DERIVE has been taken off the market and the TI-92/Voyage 200 generation of the TI-CAS handheld will not be developed further, we are looking for alternatives to be used in mathematics' education. Other than the TI-Nspire - which must be purchased by the schools and the students - there are other interesting possibilities available for free. wxMaxima is open source, WIRIS for which the online version is free in Austria and in some other countries due to general licenses, and GeoGebra, also open sourced with a newly implemented CAS, are all possible. One of these - or others? - could fill the gap left by Derive's demise.

In my lecture, I will focus on WIRIS and compare it with DERIVE and other tools. (WIRIS website: http://www.wiris.com/index.php?lang=en)

M. A. BOKHARI AND B. YUSHAU, King Fahd University of Petroleum & Minerals, Saudi Arabia
[Friday, June 26th, 11:00, B-4408]
Application of CAS to the classical definition of limit of function

The limit of a real valued function is a fundamental concept in calculus. However, most of math major and engineering students conceive the classical definition of limit as the most problematic part of calculus. They consider the $\varepsilon-\delta$ definition of limit difficult to understand. In particular, finding the largest value of δ for a given $\varepsilon>0$ in case of simple non-linear functions like $f(x)=x^2$ turns out challenging for them due to involvement of inequalities. Many students find this definition of no use and therefore, skip it. On the other hand, the role of $\varepsilon-\delta$ definition is inevitable in case of justifying some assertions like " $\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$ *does not exist*". The basic calculus books suggest the use of calculators or CAS in order to validate this type of statements because the students are usually unable to follow contra-positive arguments to justify such statements at the freshman level.

In 2006, we reformulated $\varepsilon-\delta$ definition in terms of local $(L-\varepsilon)$ approximation for a single variable function *f*. It appeared in IJMEST (V.37, No. 5, 2006, p. 515-526). Our approach for finding the value of δ is based on computing the real zeros of two functions

$$\begin{aligned} g_1(x) &= f(x)-L+\varepsilon \\ g_2(x) &= f(x)-L-\varepsilon \end{aligned} \qquad (1)$$

This, in case of a non-linear functions like $f(x)=x^n$, $f(x)=ax^n/(bx^n+c)$ or $f(x)=\cos(ax+b)$ etc, is straightforward to handle manually or with a simple scientific calculator. Nevertheless, an appropriate mathematical software is required for estimating the real zeros for several types of functions like $f(x)=(x^3-2x^2+3x+5)/(x^2+9)$ or $f(x)=\cos(x)/\ln(x-\pi+e)$ etc.

The objective of our talk is two-fold. We shall

(i) demonstrate the use of various software to the functions $g_i(x), i=1,2$ (cf (1)) by considering a variety of examples and compare their effectiveness in estimating the largest value of δ.
(ii) explain an extension of the notion of local $(L-\varepsilon)$ approximation to functions of two variables and discuss application of CAS for estimating a value of for quadric functions.

CHANTAL BUTEAU, Brock University, Canada
[Thursday, June 25th, 11:30, B-4408]
A Sustained Integration of a Computer Algebra System in University Mathematics Education at Brock University

In 2001 Brock University (Canada) launched Mathematics Integrated with Computers and Applications (MICA), a core undergraduate mathematics program developed under such guiding principles as: (1) encouraging student creativity and intellectual independence, and (2) developing mathematical concepts hand in hand with computers and applications. All traditional courses (e.g. Analysis, Algebra, etc.) were revised, and as a result, a synchronization of technology use throughout the program was carefully established. In addition, three unique project-based courses, MICA I, II, and III, were developed as a concrete implementation of the two principles above. In these courses, students learn to design, program, and use interactive computer environments (VB.net, Maple, C++) with interface in order to investigate a self-stated conjecture, concept, theorem, or real-world situation. (Examples of original students' MICA projects can be seen at www.brocku.ca/mathematics/studentprojects)

In this presentation, I will discuss the integration of CAS in the MICA program. I will exemplify how CAS is being used in a variety of courses, including in MICA II-III courses. I will conclude with a reflection on students' efficiency in using technology as a tool for learning and doing mathematics as they graduate from our program.

CHANTAL BUTEAU, NEIL MARSHALL, DANIEL JARVIS AND ZSOLT LAVICZA, Brock University, Nipissing University, Canada and University of Cambridge, UK

[Friday, June 26th, 9:30, B-4408]

Integrating Computer Algebra Systems in Post-Secondary Mathematics Education: Preliminary Results of a Literature Review

As part of an ongoing international research study that aims at analyzing the use of Computer Algebra Systems (CAS) in post-secondary mathematics instruction, we conducted in 2008 a literature review pilot study of 326 papers. The main aim of the pilot study was to inform and refine our theoretical framework, adapted from that of Lagrange et al. (2003) which they developed as a result of a large literature review of technology use in school and university mathematics education. Our revised framework will inform a more comprehensive literature review of 1,500 papers during 2009. The literature review will complement our study that also comprises a nation-wide, on-line survey of Canadian mathematics professors about their teaching practices, in comparison with results of a similar international study (United States, United Kingdom, and Hungary); and case studies of two universities (one in Canada; one in the United Kingdom) in which a mathematics department has sustained technology-related instructional change over time.

In our talk, we will discuss the results of the literature review pilot study. Several themes have emerged from the review, which will be discussed in detail in our presentation: the diverse uses of CAS, the benefits to student learning, issues of integration into mathematics learning, common and innovative uses of CAS, and the scope of CAS integration into university curricula. Our analysis suggests that, perhaps contrary to popular belief, CAS integration in tertiary mathematics teaching occurs frequently in courses for mathematics majors and not only and mainly in service courses designed for non-mathematics majors.

FRANCE CARON, Université de Montréal, Canada – **KEYNOTE SPEAKER**
[Sunday, June 28th, 10:30, B-4408]
Technology in Mathematics Education: From Meaning to Purpose

Pioneer use of CAS in mathematics education emphasised the shift from technical to conceptual work; the tool would carry out much of the technical work and multiple representations would serve to give meaning to the concepts taught. A closer look at the specificity of the techniques made possible by these tools, their observed effect on students' strategies and access to knowledge, their presence in today's mathematical practice, and the mathematics upon which they are based now call for rethinking the mathematics curriculum. This curriculum should address issues related to students' control over the use of technological tools as well as the nature and scope of problems used for instruction. In my talk, I will address this question and suggest that such curricular reflection requires revisiting the purpose of integrating technology in the teaching of mathematics and, ultimately, the purpose of mathematics education.

FRANCE CARON, DANIEL JARVIS AND KATHLEEN PINEAU, Université de Montréal, Nipissing University and École de technologie supérieure, Canada
[Sunday, June 28th, 11:15, B-4408]
Discussion: "What constitutes a good CAS task?"

The Education session at ACA 2009 filled with promising talks. The variety of speakers inspired us to organize a discussion on ***What constitutes a good CAS task?***

Interested participants will have contributed to this "finale" by sending us one or two slides that provide one example of a good CAS task with comments on

- the type and characteristics of students with which this task has been (or could be) used
- the wording of the task
- what it requires the students to do (in terms of math and CAS functionality)
- why it qualifies as a "good CAS task"

A quick presentation of the assembled slides will serve to launch the discussion.

BRUCE CHAR, JEREMY JOHNSON AND DAVID AUGENBLICK, Drexel University, USA
[Thursday, June 25th, 12:00, B-4408]
Using Maple and Maple TA in a course about technical computing

Engineering Computation Lab is a year-long one credit/term sequence taught to approximately 700 Engineering freshmen at Drexel University. It combines in-class collaborative lab work in small sections, with on-line homework with automated feedback and grading. Typically 75% of the contact time is spent in hands-on active learning.

Distinctive elements include: a) A course platform of an interactive interpreted system with extensive built-in technical functionality (Maple Computer Algebra System) rather than a "generic" compiled language. This means that students can quickly get useful results, rather than going through the learning curve with "hello world" tasks. Over time, the course makes a transition from "calculator-like" sequences in a GUI, to scripting and then procedures. b) Emphasis on computing for technical/scientific problems: simulation and computational exploration using numerical computation and visualization, symbolic computation for calculus- or algebra-based model-derivation. c) Sensible use of technical computing is the primary instructional objective. The course uses the science and mathematics the students have already encountered in other courses. We thus have time to talk about computing concepts (data structures, control structures, procedures and types) without a heavy science/math pedagogical agenda. This introduces the terminology and conceptual framework that should allow better transfer of this knowledge to other programming languages and systems.

Standard educational IT (CMS, mailing lists, wiki) is used extensively. Distinctive IT use includes laboratories equipped for support of collaborative group work and small-scale coaching sessions, and use of an on-line quiz/exam system (Maple TA) that delivers staff-authored individually-generated versions of problems to students, with immediate feedback. These elements have allowed us to run the course and its trailer sections using a small number of senior staff with peer tutor undergraduate assistants and graduate student TAs. While not occupying the primary focus, the non-floating point features of computer algebra systems (exact solution and calculus operations, extended precision numerics, list processing, and formula-based visualization) are used as part of the every day coursework for exploring and solving technical problems.

KEN COLLINS, Charlotte Latin School, USA
[Friday, June 26th, 11:30, B-4408]
Analysing Power Series using Computer Algebra and Precalculus Techniques

Many calculus students have difficulty with power series questions. This session will examine explorations of power series using the TI-89 and precalculus techniques that help students understand how to generate and use power series to represent functions. Classroom ready handouts will be provided.

Areas of mathematics education involved with this paper: problem solving, use of technology, classroom practices, rich learning tasks, and improving curriculum.

The goals of this session are to demonstrate how we can use calculator technology, computer algebra, and precalculus mathematics to generate power series that model functions. We will develop power series models that are normally introduced only at the calculus level. We believe that many calculus students have difficulties with power series because they must learn power series and calculus techniques simultaneously. If they had a better understanding of power series before applying them in calculus they would have more success learning this topic. This session will share several handouts that use calculator explorations of power series. We will discuss typical student questions that arise from these investigations. We will describe how to generate additional investigations that are suitable for your classes. We encourage participants to actively question and share.

JAIME CURTS, University of Texas Pan American, USA
[Thursday, June 25th, 16:00, B-4408]
Teaching Principal Component Analysis in Minitab®

The purpose of this paper is to introduce the logical and arithmetic operators and simple matrix functions of Minitab® –a well-known software package for teaching statistics- as a computer-aid to teach Principal Components Analysis (PCA) to graduate students in the field of Education.

PCA, originally proposed by Pearson (1901) is a mathematical technique –a vector space transform- that has its roots in linear algebra and in statistics. Its main purpose is to reduce a correlated multidimensional data set to an uncorrelated lower dimensional space with maximum variance. PCA concepts can be a roadblock for non-mathematical oriented students, since statistical definitions (i.e., variance-covariance, correlation) need to be connected to matrix algebra (eigenvectors of a variance-covariance matrix) and to graphical vector representation (including matrix rotation).

Effective teaching of PCA requires students to develop a “feeling” for the intuitive sense of eigenvalues and eigenvectors and Minitab® provides a flexible vector-based environment and tools which students can use for meaningful learning of such concepts.

Though MINITAB by no means falls under the definition of a computer algebra system (CAS), it is well suited –among other functions- to manipulate, calculate, transform, and save data or matrices (transpose, inverse, eigen-analysis and matrix arithmetic) and has been reported to be successful teaching linear algebra (Greenwell, 1985). Following Güyer’s general taxonomy of computer aided mathematics education software (Güyer, 2008), Minitab® is here considered as a general purpose software that can efficiently be used as a teaching tool and may interact with other CAS. It is not as sophisticated as Maple® or Mathematica®, but the purpose here is to illustrate how to teach PCA by manipulating data sets and taking advantage of the software editing facilities (Bassett, Brooks, & Morgan, 1995).

This presentation will also show how to use the graphical capabilities of Minitab® and illustrate the bi-plot graphic display of matrices with application to PCA (Gabriel, 1971). Bi-plots can be used to support students’ conceptual and cognitive difficulties with the geometrical interpretation of eigenvectors and eigenvalues. Earlier, Gould (1967) suggested to consider a matrix of coordinates of points in space and interpret the eigenvalues and associated functions as geometric properties of the arrangements of these points.

Finally this paper wants to illustrate Minitab® as a learning tool to support graduate students in the field of Education in their conceptual understanding of reducing data dimensionality and other multivariate methods. These students are commonly introduced to PCA as one of the most commonly used exploratory data reduction procedure used in educational research at the same time they experience the analysis and computation of large number of variables. For such purpose ample examples of PCA guides to educational research exist (ex., Osborne & Costello, 2004; Pohlmann, 2004) and its application is evidenced by the wide range of educational research studies as exemplified by numerous perceived self-efficacy beliefs studies (e.g. Curts, Tanguma, & Peña, 2008; Dellinger, Bobbett, Oliver, & Ellet, 2008).

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JAMES DAVENPORT, University of Bath, UK
[Saturday, June 27th, 11:30, B-4408]
A Comparison of Equality in Computer Algebra and Correctness in Mathematical Pedagogy

Joint work with Russell Bradford (Bath) and Chris Sangwin (Birmingham).

How do we recognize when an answer is "right"? This is a question that has bedevilled the use of computer systems in mathematics (as opposed to arithmetic) ever since their introduction. A computer system can certainly say that some answers are definitely wrong, in the sense that they are provably not an answer to the question posed. However, an answer can be mathematically right without being pedagogically right. Here we explore the differences in the context of computer marking of traditional homework and show that, despite the apparent distinction, it is possible to make many of the differences amenable to formal treatment, by asking "under which congruence is the pupil's answer equal to the teacher's?"

Jiro Doke, MathWorks, USA
[Saturday, June 27th, 12:00, B-4408]
Using Symbolic Math in Engineering Education

MATLAB is used extensively in engineering schools worldwide. It's numeric foundation, algorithm development and data analysis tools, and domain specific add-ons for control system design and signal processing make it a preferred tool for teaching and learning engineering concepts. Symbolic Math Toolbox extends the numeric capabilities of MATLAB with tools for analytical modeling and calculation management. Developing analytical models helps engineering students better understand the behavior of the systems they study, since the underlying parameters driving system behavior are transparent.

This lecture will use specific application examples to show how symbolic math could be in System Dynamics courses, for plant modeling tasks, and Aerospace Design courses, for aircraft modeling tasks. Emphasis will be placed on how symbolic math complements numeric computations.

Wade Ellis and William C. Bauldry, West Valley College and Appalachian State University, USA
[Friday, June 26th, 10:30, B-4408]
Using and Creating A-C-R Documents for Mathematics Instruction with Computer Algebra

We will present the Action-Consequence-Reflection (ACR) paradigm for using computer algebra didactically. First, examples of available documents for both hand-held and desktop computer algebra systems will be shown along with methods for their use in the classroom. Second, we will illustrate how easy it now is to make ACR documents with a TI- Nspire CAS and Maple 13. Our sample ACR documents will be chosen from topics relevant to precalculus up to real and complex analysis. We'll end the session with a discussion of classroom uses of ACR documents.

Philippe Etchecopar, Jordi Nadal and Jean-Philippe Villeneuve, CEGEP de Rimouski, Canada
[Thursday, June 25th, 11:00, B-4408]
Integrating CA in Modelling-Simulation Approaches in the Mathematics Courses of the Science Programs at the CEGEP of Rimouski

The exponential development of technology has deeply transformed work practices in the scientific world. These new work methods, based on modelling of scientific phenomenon and their simulations, are being developed in many disciplines and in doing so, are increasing the importance of mathematical content in these disciplines.
To better prepare students for scientific studies in this context, fifteen years ago the department of mathematics of the CEGEP of Rimouski introduced Maple and MatLab in a modelling-simulating approach in all the mathematics courses of their scientific programs.
Based on examples from differential and integral calculus, we will address some of the issues encountered in the past 15 years: changes in curriculum, the use of technology, interdisciplinary approach combining mathematics with physics and biology as well as the importance of responsible citizenship.

DANIEL JARVIS, Nipissing University, Canada
[Saturday, June 27th, 16:00, computer lab B-2404]
GeoGebra Exploration (Workshop)

Come experiment with the GeoGebra software. An informal drop-in workshop will be offered for those participants interested in exploring GeoGebra and sharing their knowledge of, and experiences with, this popular open-source software.

DAVID J. JEFFREY, University of Western Ontario, Canada
[Saturday, June 27th, 11:00, B-4408]
Getting from x to y without crashing

When teaching with technology, we want to teach mathematics, but we cannot avoid teaching software as well. In a similar way, a car is a means of getting from A to B, but you cannot make the journey without a driver's license. I discuss my experience of getting from x to y using Maple and Matlab. In both cases, I have collected the places where students most often crash and I have developed teaching material to set up buffers before they arrive. For example, in Maple there are differences between expressions and functions; in Matlab [1,2]/[4,5] is different from [1,2]./[4,5].

PATRICIA LEINBACH AND CARL LEINBACH, Adams County and Gettysburg College, USA
[Thursday, June 25th, 14:00, B-4408]
Using Forensic Investigations and CAS to Motivate Student Interest in Mathematics I and II

Patricia Leinbach, Coroner of Adams County, Pennsylvania (retired), and Carl Leinbach propose to give two, consecutive sessions in the Education session of ACA 2009 on the topic of “Using Forensic Investigations and Computer Algebra to Motivate the Study of Mathematics.”

The first session will deal with the layout of a crime scene based on Patricia’s experience and use actual scene materials to the extent that they do not compromise individual’s rights to privacy to show where mathematics is used and also the type of mathematics. The second session will deal with the student use of CAS to analyse the material and data gathered at the scene of the coroner’s investigation.

Session I [Thursday, June 25th, 14:00, B-4408]

It is an unfortunate occurrence, although not uncommon, that students fail to appreciate the importance of mathematics during their student career. Later in life, either through experiences in their jobs or during the further study of a subject of interest, they regret that they did not pay more attention during their mathematics classes. Another deterrent to their learning is the seeming emphasis on techniques and not the reasoning and logic behind the techniques. The purpose of these sessions is to provide a motivation for learning mathematics and the use of CAS to provide a shift in the emphasis that students perceive and dislike so much about mathematics teaching.

In this portion of our two session presentation, we will carefully examine an actual crime scene and the opportunities for using mathematics to gain a forensic (legally accepted) insight into the evidence that is found on the scene. In addition we will look at the type of mathematics that is required to gain this insight and discuss the student level at which this mathematics may be presented. We will also examine the role of the CAS in the analysis of the data and evidence collected at the crime scene. The major portion of this session will be presented by Pat Leinbach and the Coroner's perspective will be emphasized.

Session II [Thursday, June 25th, 14:30, B-4408]

This session will be a continuation of the first session on this topic. It will have a much heavier emphasis on the mathematics and use of the CAS in analysing the data and evidence from the crime scene. It will also show ways that students can create their own "crime scene" in a mathematics laboratory session. During the discussion section the presenters will respond to questions concerning the appropriateness and usefulness of incorporating these materials into a mathematics classroom and resources for obtaining ideas and materials for further investigations.

The CAS's to be used in this session are DERIVE 6 and TI N-spire.

ROBERT LOPEZ, Rose-Hulman Institute of Technology, USA and Maple Fellow - Maplesoft, Canada
[Friday, June 26th, 12:00, B-4408]

Resequencing of Skills and a CAS You Don't Have to Teach

Resequencing of skills – stressing concepts first and delaying skill development by relying on a CAS – is an idea approximately 20 years old. This talk will revisit this pedagogical technique, using examples implemented in Maple, a CAS that doesn't have to be taught.

Intuition and the "big picture" are addressed by using the computational and visualization power of the CAS. The concept is experienced, its connection to other parts of the curriculum explored, the correctness of solutions verified, all within the computational framework of the CAS.

Then the steps of applicable algorithms are implemented in the CAS. Here, it is essential that the software tool be easy to use, flexible, and "in sync" with the flow of the mathematical calculations. After a bit of practice with this approach, students can be set to acquire those manipulative skills necessary for the overall curriculum.

This approach to the pedagogical implementation of a CAS in the classroom will be illustrated with several examples. From these experiences, participants can determine the viability and soundness of the approach. And they can observe the added clarity that derives from the syntax-free environment of Maple's point-and-click paradigm.

This point-and-click paradigm is sufficiently transparent that an instructor can simply use it as appropriate in a lesson, and students who observe its use will be able to imitate and extend the calculational approach without specific instruction on the use of the software tool. The GUI devices built into the software are natural enough that they require little or no explanation. This simplicity in the use of the tool means implementing the steps of a calculation in the software will not seem like more work for the student, and the instructor will not be inclined to neglect an appropriate amount of drill-and-practice that leads to the mastery of skills necessary for success in mathematics.

M. B. MONAGAN AND J. F. OGILVIE, Simon Fraser University, Canada and Universidad de Costa Rica, Costa Rica
[Thursday, June 25th, 15:00, B-4408]
Teaching and learning mathematics with symbolic computation

Although for one or two decades mathematical software for symbolic computation has been employed extensively in the teaching of university mathematics, each respective course typically emulates a traditional course without computers in treating a particular topic such as differential calculus or linear algebra. We advocate an holistic approach to the teaching and learning of mathematics involving intensive use of contemporary software not only for pedagogical purposes but especially for implementation by the users of mathematics. By expecting a learner to employ computer software for almost all mathematical operations, an instructor can emphasize the mathematical concepts and principles, involving formal definitions, algebraic derivations, numerical examples and especially graphical illustrations and constructions, and then develop the implementation with selected software. In this way not only does a student acquire a profound understanding of those principles – and limitations of the software, but he or she becomes proficient in applying the software for real problems of a scale that would be impracticable in manual work. By eliminating practically all repetitive drill and practice that is irrelevant when the computer undertakes the calculations, an instructor becomes able to cover a much enhanced range of topics within a given duration; for instance, all material – from differential, integral and multivariate calculus, linear algebra and differential equations to probability, statistics and data analysis that typically occupies six or more semester courses – might be covered within three semesters at a typical pace. That content is all that a student is likely to need for a technical career after completing an undergraduate programme in science and engineering. We argue that contemporary students of engineering and science who are not so equipped with a working knowledge of symbolic mathematical software are not being prepared properly for a technical career.

> *"The human mind is never performing its highest function when it is doing the work of a calculating machine."* – Lord Kelvin

In this lecture we present examples of topics of which we demonstrate some benefits of teaching with mathematical software.

ALEKSANDR MYLLÄRI AND TATIANA MYLLÄRI, University of Turku and Åbo Akademi University, Finland
[Thursday, June 25th, 16:30, B-4408]
CAS in Teaching Basics of Statistical Learning

Modern computer algebra systems not only make calculations (analytic and numeric) easy, but also have good visualization facilities. Visual demonstrations provide convenient way to demonstrate the work of the algorithms and help students to understand them. We consider the problem of binary classification. Support Vector Machines (SVMs) are attractive from the educational point of view since it is easy to introduce them gradually: from simple perceptron to more and more advanced classifiers. We start with the Rosenblatt's perceptron – simple binary classifier for linearly separable cases – and generalize it to maximum margin classifier; then we introduce kernel trick that allows generalization to nonlinearly separable cases, and finally accept misclassifications on the training stage. We model the work of the Rosenblatt's perceptron and simple SVMs using Mathematica 6 and Maple 12. Constructed models are used in the introductory courses on SVMs and Statistical Learning.

YVAN SAINT-AUBIN, Université de Montréal, Canada – **KEYNOTE SPEAKER**
[Friday, June 26th, 8:45, B-4408]
Animations in Mathematics

Essentially all computer algebra systems offer some graphic primitives or functions. So it is not surprising that some also offer the possibility of doing animations. With progress in software and some experience, the time cost of building animations has decreased to an acceptable level.

Animations have therefore the potential to become a common pedagogical tool for the first time in the history of mathematics. Obvious uses are the depiction of phenomena that evolve in time: the motion of a pendulum, the evolution of populations in a prey-predator model, etc. But animations can also be used to clarify mathematical ideas or constructions where time plays no role. I will present and discuss some animations of this latter type.

KARSTEN SCHMIDT, Schmalkalden University of Applied Sciences, Germany
[Sunday, June 28th, 9:00, B-4408]
Mathematics Education with a Handheld CAS – The Students' Perspective

Between 1999 and 2003 a project was carried out in eight upper secondary schools in Thuringia (one of the 16 German states) to investigate the effects the use of CAS technology in mathematics education has on math skills. In 2002 a survey of all students (N = 1014) in the project schools took place. The main part of the one-page questionnaire consisted of eight statements – seven related to the effects of using a handheld CAS in math and science lessons and one general statement about math lessons.

Since 2003 each school in Thuringia can decide if a handheld CAS is used in mathematics education or not. One year later more than a quarter of all Thuringian upper secondary schools used CAS in mathematics classes. The 2002 survey was carried out again in 2005, this time with 1679 students.

Results of the two surveys will be analyzed and compared. Special focus will be on the question if certain characteristics of the students (e.g. their gender or how good they are in mathematics) influenced their answers.

DENIS TANGUAY AND ANDRÉ BOILEAU, Université du Québec à Montréal (UQAM), Canada
[Sunday, June 28th, 9:30, B-4408]
Using CAS in Letter-Symbolic Algebra at the Secondary Level : a Classroom Activity

Computer Algebra Systems (CAS) are becoming more and more important in high school, cegeps and university maths courses. Whether they involve graphing calculators or software such as Maple or Derive, their use is often confined to teaching situations where functions are at stake. In such instances, we discern three main pedagogical motivations :

- the idea that through the use of CAS, one can give students access to sophisticated situations and modellings otherwise too complex, the needed computations lying beyond the scope of students' techniques ;
- the related and more general idea that by relieving students of the (tedious) traineeship of computational techniques, more time can be devoted to conceptual apprenticeships ;
- the idea that these complex functional situations can then be studied through semiotic representations in various registers (Duval, 1993), giving rise to work involving conversion/coordination between these registers — researchers in math education being more and more convinced that this type of work is the basis for solid conceptualization.

At the secondary level, this third idea prevails, and graphing calculators in math classes are used mainly in algebra courses, as a tool to go back and forth between algebraic expressions of a given function, its table of values and its graph. The aim of the APTE team is to extend this usage by employing CAS calculators in order to help students in their construction of meaning and conceptualization, within the more literal-symbolic segment of secondary level algebra, apart from any functional consideration. The team has thus designed classroom activities (i. e. consistent and connected sequences of tasks) targeting an apprenticeship of techniques in algebra (at the 3rd and 4th secondary level), such as factorization and expansion, equation solving, substitution, while fostering the conceptual (theoretical) thinking for such notions as :

- equivalence of expressions;
- domain of validity for an expression or for an equivalence;
- solution set of an equation or system of equations;
- the distinction equation-identity, etc.

In this communication, we will present one of these activities, with the relevant work and productions from the students with which it has been experimented. We will discuss some aspects of its design which we evaluate as important, in particular regarding the co-emergence of technique and theory and their mutual interactions (Kieran & Drijvers, 2006): going back and forth from paper-and-pencil work to CAS work, comparison between standard algebraic syntax and CAS syntax, triggering use of the unexpected/startling CAS-output, conjectures, justifications of the conjectured formulae, large group discussions, role of the teacher...

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YUZITA YAACOB, MICHAEL J. WESTER AND STANLY STEINBERG, National University of Malaysia, Malaysia and University of New Mexico, USA
[Saturday, June 27th, 10:30, B-4408]
Developing an Automated Learning Assistant for Vector Calculus

This paper presents a prototype of a computer learning assistant ILMEV (Interactive Learning - Mathematica Enhanced Vector calculus) package with the purpose of helping students to understand the theory and applications of integration in vector calculus. ILMEV is built on the important pedagogical concepts (interactivity, visualization and experimentation), computer based learning theory (behaviorism, cognition and humanism), simple logical steps in problem solving (with some explanations), and an easy to use interface. Vector calculus was chosen as the subject area because even mathematically talented students in engineering and science find this subject difficult. No computer algebra system has algorithms to automatically solve all but the most elementary problems of this type that appear in textbooks. To overcome this, we implemented a prototype of ILMEV, which can compute closed form solutions to many two dimensional textbook examples without substantial user intervention. ILMEV succeeds because it contains algorithms for reducing the integrals appearing in vector calculus to sums of iterated integrals. The typical presentation of this material in textbooks was reorganized and subsequently a model was created which presented an overview of many of the important integrals that appear in vector calculus. This model provides a framework for developing the complete content of ILMEV and other vector calculus packages.

2. Interaction Between Computer Algebra and Interval Computations

org : Walter Kraemer, Bergische Universitaet, Germany
Markus Neher, University of Karlsruhe, Germany
Evgenija D. Popova, Bulgarian Academy of Sciences, Bulgaria

Room: B-2624

Schedule

Thursday June 25th

10:30 – 11:00	KYOKO MAKINO Taylor model based optimization
11:00 – 11:30	MARTIN BERZ Taylor model based methods for ODEs
11:30 – 12:00	MARKUS NEHER Verified integration methods for ODEs
12:00 – 12:30	ALEXANDER WITTIG Classification of all high period periodic points of maps and flows
14:00 – 14:30	WALTER KRAEMER Symbolic-numeric computations using arbitrary precision intervals
14:30 – 15:00	JORIS VAN DEUN Hypergeometric functions accurate to the last digit
15:00 – 15:30	EKATERINA AUER Applications of verified methods in biomechanics

Abstracts

EKATERINA AUER, University of Duisburg-Essen, Germany
[Thursday, June 25th, 15:00, B-2624]
Applications of verified methods in biomechanics

In the recent years, verified methods have been applied in engineering to propagate uncertainty, for example, measurement uncertainty, through a given system and to compute its parameter sensitivities. The measurement uncertainty becomes especially problematic in biomechanics where the presence of living subjects prohibits frequent use of some more precise methods that have adverse health effects. One of the drawbacks of verified solutions to this problem from the point of view of mechanics is that they use derivatives which are in general not available inside numerical modeling and simulation software.

The problem of obtaining derivatives can be solved, for example, by using algorithmic differentiation implemented through overloading. However, this presupposes that the code we use does not contain conditional expressions that depend on their argument, that is, directives of the form IF x<0 THEN f(x)=f1(x). Generally, this too rigid restriction obstructs the applicability of verified methods. Recently, algorithmic differentiation tools have been developed that can handle conditional expressions for floating-point- based codes (e.g. CppAD, http://www.coin-or.org/CppAD/). The task now is to adjust them to interval-based data types.

However, this task is not as straightforward as it might seem. An interval comparison operator might have a number of semantically different definitions. The one most suitable in our situation can take not only true or false as its value. There is also a third case to consider where we cannot tell how one interval compares to another, the so-called `maybe case'. In this talk, we will discuss possible solutions to the problem of differentiating piecewise functions in interval-based implementations. Moreover, we will give an idea of how computer algebra methods can be applied in a general biomechanical context.

Martin Berz, Michigan State University, USA
[Thursday, June 25th, 11:00, B-2624]
Taylor model based methods for ODEs

Taylor models combine the advantages of numerical methods and algebraic approaches of efficiency, tightly controlled recourses, and the ability to handle very complex problems with the advantages of symbolic approaches, in particularly the ability to be rigorous and to allow the treatment of functional dependencies instead of merely points. The resulting differential algebraic calculus involving an algebra with differentiation and integration is particularly amenable for the study of ODEs and PDEs based on fixed point problems from functional analysis. We describe the development of rigorous tools to determine enclosures of flows of general nonlinear differential equations based on Picard iterations. Particular emphasis is placed on the development of methods that have favorable long term stability, which is achieved using suitable preconditioning and other methods. Applications of the methods are presented, including determinations of rigorous enclosures of flows of ODEs in the theory of chaotic dynamical systems.

Walter Kraemer, University of Wuppertal, Germany
[Thursday, June 25th, 14:00, B-2624]
Symbolic-numeric computations using arbitrary precision intervals

The computer algebra system Maple allows to control the rounding mode of (arbitrary precision) arithmetic operations. As we will see, using this feature naively may result in unexpected results. There are also situations in which the evalr command is not faithful to the power set model for interval arithmetic.

However, the interval package intpakX (available as a Maple Power Tool since 2002) may be used to perform arbitrary precision interval operations in a safe way. In the first part of the talk we give a summary of the most interesting features supported by intpakX. The second part we show how symbolic-numeric computations may be used to solve several sample problems with guaranteed results (the computed results may be interpreted as computer-assisted proofs of mathematical properties of the problems under consideration). Different approaches to the reliable computation of orbits of a chaotic dynamical system will be discussed in some detail.

KYOKO MAKINO, Michigan State University, USA
[Thursday, June 25th, 10:30, B-2624]
Taylor model based optimization

A Taylor model of a smooth function *f* over a sufficiently small domain *D* is a pair (*P,I*) where *P* is the Taylor polynomial of *f* at a point *d* in *D*, and *I* is an interval such that *f* differs from *P* by not more than *I* over *D*. As such, they represent a hybrid between numerical techniques for the interval and the coefficients of *P* and algebraic techniques for the manipulation of polynomials. A calculus including addition, multiplication and differentiation/integration is developed to compute Taylor models for code lists, resulting in a method to compute rigorous enclosures of arbitrary computer functions in terms of Taylor models. The methods combine the advantages of numeric methods, namely finite size of representation, speed, and no limitations on the objects on which operations can be carried out with those of symbolic methods, namely the ability to treat functions instead of points and making rigorous statements.

We show how the methods can be used for the problem of rigorous global search based on a branch and bound approach, where Taylor models are used to prune the search space and resolve constraints to high order. Compared to other rigorous global optimizers based on intervals and linearizations, the methods allow the treatment of complicated functions with long code lists and with large amounts of dependency. Furthermore, the underlying polynomial form allows the use of other efficient bounding and pruning techniques, including the linear dominated bounder (LDB) and the quadratic fast bounder (QFB).

MARKUS NEHER, University of Karlsruhe, Germany
[Thursday, June 25th, 11:30, B-2624]
Verified integration methods for ODEs

Verified integration of ODEs. The numerical solution of initial value problems (IVPs) for ODEs is one of the fundamental problems in computation. Today, there are many well-established algorithms for approximate solution of IVPs. However, traditional integration methods usually provide only approximate values for the solution. Precise error bounds are rarely available. The error estimates, which are sometimes delivered, are not guaranteed to be accurate and are sometimes unreliable.

In contrast, verified integration computes guaranteed bounds for the flow of an ODE, including all discretization and roundoff errors in the computation. Originated by Moore in the 1960s, interval computations are a particularly useful tool for this purpose.

Dependency Problem and Wrapping Effect. Unfortunately, the results of interval arithmetic computations are sometimes affected by overestimation, such that computed error bounds are over-pessimistic. Overestimation is often caused by the \textit{dependency problem}, which is the lack of interval arithmetic to identify different occurrences of the same variable. For example, \$x - x > = 0\$ holds for each \$x \in [1,2]\$, but \$\bmx - \bmx\$ for \$\bmx = [1,2]\$ yields

[-1,1]$. A second source of overestimation is the <i>wrapping effect</i>, which appears when intermediate results of a computation are enclosed into intervals.

Overestimations due to wrapping are one of the major problems in the interval arithmetic treatment of ODEs. In verified integration, overestimation may degrade the computed enclosure of the flow, enforce miniscule step sizes, or even bring about premature abortion of an integration.

Taylor Models. Berz and his co-workers have developed Taylor model methods, which combine interval arithmetic with symbolic computations. For the verified integration of IVPs, Taylor models supply a comprehensive variety of applicable enclosure sets for the flow, which is an effective means for reducing wrapping.

In our talk, we present Taylor model methods for the verified integration of ODEs and compare them with well-known interval methods for this task. Numerical examples for linear and nonlinear ODEs are given.

JORIS VAN DEUN, University of Antwerpen, Belgium
[Thursday, June 25th, 14:30, B-2624]
Hypergeometric functions accurate to the last digit

We present an efficient algorithm for the validated high precision computation of real continued fractions, accurate to the last digit. The algorithm proceeds in two stages. In the first stage, computations are done in double precision. A forward error analysis and some heuristics are used to obtain an a priori error estimate. This estimate is used in the second stage to compute the fraction to the requested accuracy in high precision (adaptively incrementing the precision for reasons of efficiency). A running error analysis and techniques from interval arithmetic are used to validate the result.

As an application, we use this algorithm to compute Gauss and confluent hypergeometric functions when one of the numerator parameters is a positive integer.

ALEXANDER WITTIG, Michigan State University, USA
[Thursday, June 25th, 12:00, B-2624]
Classification of all high period periodic points of maps and flows

"We present an algorithm for a rigorous, global periodic point finder. This algorithm allows us to rigorously identify small enclosures of fixed and periodic points of a sufficiently smooth map $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ in a given region $K\subset\mathbb{R}^n$. If the derivative of f is known, we can furthermore verify uniqueness of the fixed or periodic points in each enclosure.

We then proceed to present an implementation of this algorithm in Taylor Model arithmetic using COSY INFINITY. The application of this implementation to the locally hyperbolic Hénon map demonstrates the power of the Taylor Model approach. Taylor Models combine the speed of numerical methods with the low overestimation of symbolic computation, thus avoiding the problems introduced by other verified numerical methods such as interval arithmetic. This allows us to compute small enclosures of all periodic points with period 13 in the attractor of the Hénon map.

3. APPLICATIONS AND LIBRARIES DEVELOPMENT IN DERIVE

org : José Luis Galán García, Universidad de Málaga, Spain
Pedro Rodríguez Cielos, Universidad de Málaga, Spain
Gabriel Aguilera Venegas, Universidad de Málaga, Spain
Josef Böhm, Austrian Center for Didactics of Computer Algebra, Austria

Room: B-3432

Schedule

Friday June 26th

8:30 – 9:00	JOSEF BÖHM CAS Tools for Exercising
9:00 – 9:30	EUGENIO ROANES LOZANO AND FRANCISCO A. GONZÁLEZ REDONDO An implementation on the Mayan numbering system in DERIVE
9:30 – 10:00	KARSTEN SCHMIDT Making Life in an Introductory Linear Algebra Course Easier with Derive
10:30 – 11:00	ALFONSA GARCÍA, FRANCISCO GARCÍA, GERARDO RODRÍGUEZ, AND AGUSTÍN DE LA VILLA Toolboxes with DERIVE
11:00 – 11:30	MICHEL BEAUDIN Another Look at a Trusted Mathematical Assistant
11:30 – 12:00	G. AGUILERA, J. L. GALÁN, M. Á. GALÁN, Y. PADILLA, AND P. RODRÍGUEZ Generating random samples from continues and discrete distributions with Derive

Abstracts

G. AGUILERA, J. L. GALÁN, M. Á. GALÁN, Y. PADILLA, AND **P. RODRÍGUEZ**, University of Málaga, Spain
[Friday, June 26th, 11:30, B-3432]

Generating random samples from continues and discrete distributions with Derive

In this talk we will introduce the utility file ***RandomDistributions.mth***. This file has been developed for generating random values from main continuous and discrete distributions.

The programs contained in the file can be grouped within the following blocks:

- **Random values from uniform distribution**: the program ***RandomUniform*** returns an uniform random sample between 0 and 1. This program is the base of the other generations. Different algorithms have been used to develop this program in order to improve the Derive's build-on function to generate samples from a continue uniform random distribution.

- **Random values from discrete distributions**: some generic algorithms for any discrete distribution have been implemented as well as specific algorithms for some discrete distributions (Uniform, Poisson, Binomial, Geometric, Negative Binomial, …).
- **Random values from continuous distributions**: specific algorithms for main continuous distributions have been implemented (Uniform, Exponential, Normal, Lognormal, Cauchy, Chi-square, Student's t, F, Gamma, Beta, …).
- **Graphical approach**: a program to plot the obtained samples together with the density function has been developed. With these drawings we can check graphically if the generated samples fit the distributions. The use of this utility file is useful for simulating any process which follows a specific distribution.

MICHEL BEAUDIN, École de technologie supérieure, Canada
[Friday, June 26th, 11:00, B-3432]
Another Look at a Trusted Mathematical Assistant

From the DERIVE user manual (version 3, September 1994), we can read the following: "Making mathematics more exciting and enjoyable is the driving force behind the development of the DERIVE program". In this talk, we will try to show how some mathematical concepts, studied by engineering students at university level - differential equations, multiple variable calculus, systems of non linear equations -, can be easily illustrated by DERIVE. Some will object that any other CAS could do the same: well, this is probably true but, according to us, not as quickly and naturally: "To accomplish this DERIVE not only has to be a tireless, powerful and knowledgeable mathematical assistant, it must be an easy, natural, and convenient tool". Consequently, time can be spent to prove some theorem or formula and the computer algebra system helps to reinforce the mathematical concepts. Our examples will also make use of new features added in the latest version of DERIVE (version 6.10 released in October 2004); features that were not exploited as should be - DERIVE has never been enough documented. But we are still convinced that Derive 6 was "Far too good just for students" (http://www.scientific-computing.com/scwmarapr04derive6.html).

JOSEF BÖHM, Technical University and Austrian Centre for Didactics of Computer Algebra, DUG, Austria
[Friday, June 26th, 8:30, B-3432]
CAS Tools for Exercising

There is no doubt that even in times of CAS a certain amount of manipulating skills in various fields of math education is still necessary.

Students need more or less exercising for mastering expanding and factorizing expressions, finding GCD and LCM, solving triangles, applying differentiation and integration rules to name only some of the fields where training of skills might be useful.

We present a respective library developed in Derive reaching from set theory to calculus, which can support the students (and teachers as well) offering random generated problems together with the respective solutions.

Alfonsa García, Francisco García, Gerardo Rodríguez, and **Agustín de la Villa**, Universidad Politécnica de Madrid, Universidad de Salamanca, Universidad Pontificia Comillas, Spain
[Friday, June 26th, 10:30, B-3432]
Toolboxes with DERIVE

The European Area of Higher Education implies a profound change in the Spanish university. We are heading towards a competency-based teaching and a learning model with greater autonomy for the student, who becomes the centre of the educational model. New technologies can play an important role in this new scenario. This paper suggests a "new" possibility in the use of new technologies: The design of a "toolbox" which could be used later on by the student when needed in other subjects of their curriculum or in their careers. This “toolbox” will contain instructions, which may either be suggested by the teacher or developed by the student as being interesting and convenient so as to explore the mathematical concepts associated with them.

There are two examples of possible "toolboxes." In the first one, instructions are related to **geometrical aspects** of the plane which are studied in high school. The instructions are concerning with drawings, equations of different geometric objects and some distance between the quoted objects.

In the second one, a toolbox is built for the subject **Calculus of a single variable** in engineering studies. In this toolbox there can be instructions about complex numbers to express them in their different expressions, calculate their power, roots, etc. It also incorporates the instructions that enable to analyze the concepts of differential calculus such as the tangent, the study of increasing and convexity, etc.

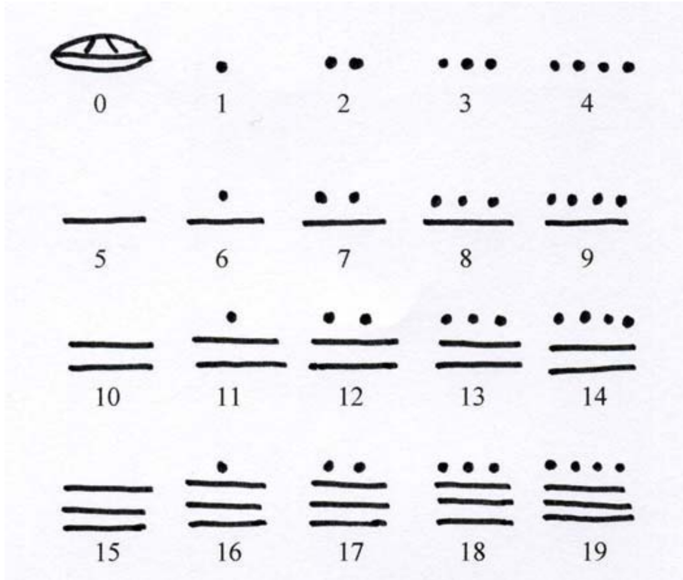
About integral calculus several tools can be included for the geometric applications. Also procedures for the trapeze method and the Simpson method are implemented.

In the study of approximate methods of solving equations we can encourage students to use algorithms NEWTON and FIXED_POINT, which DERIVE has incorporated, and design a procedure for the method of the bisection of the interval.

Eugenio Roanes Lozano and **Francisco A. González Redondo**, Universidad Complutense de Madrid, Spain
[Friday, June 26th, 9:00, B-3432]
An implementation on the Mayan numbering system in DERIVE

The Mayan number system is a base *20*, positional (to be read from top to bottom, not from left to right) system that makes use of a symbol representing zero. It has slightly different variations when used for counting days (in religious and astronomical contexts).

Therefore, *20* symbols are needed to represent *0,1,2,…,19*. Of these, the positive ones were represented using dots (the value of each dot is *1*), and horizontal segments (the value of each segment is *5*), while the zero was denoted by a shell (see figure below).



If a number is greater than *20*, the symbols corresponding to units, twentieths, *400's*, *8000's*… are stacked from bottom (units) to top in pure base *20*, while in our decimal system, the different orders correspond to tenths, hundreds, thousands, etc.

We can choose the input and output bases in *DERIVE* in *Options > Mode Settings* from *2, 8, 10* and *16*. Therefore we have implemented a procedure that allows to convert numbers between any bases, and that returns the output in (row) vector style. We have implemented another procedure that builds the *20* Mayan symbols for *0,1,2,…19* (making use of the DISPLAY command). Finally, another procedure (denoted Maya), that uses the previously mentioned procedures, converts any number from base *10* to base *20* and represents it in the Mayan numbering system. These procedures only make use of the standard DERIVE commands.

For instance, for *2821* we would obtain:

```
#10:                maya(2821) =
```

(note that *2821 = 6 × 20 × 20 + 16 × 20 + 1*).

We do not know of any other similar implementations in CASs, apart from [1] (that uses special facilities for inserting graphics) and the similar [2], by the same authors of this paper, but written in *Maple*.

We believe this is an interesting example of synergy among different branches of knowledge (Mathematics, History of Mathematics and Computer Science), that can increase the interest of the students for different topics.

Acknowledgments

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Bibliografia

[1] URL: http://www.mapleprimes.com/forum/convertmayannumber

[2] E. Roanes Lozano, F.A. Gonzalez-Redondo: Implementación del sistema de numeración maya en *Maple*: una experiencia interdisciplinar. Bol. Soc. Puig Adam 83 (2009). To appear.

Karsten Schmidt, Schmalkalden University of Applied Sciences, Germany
[Friday, June 26th, 9:30, B-3432]
Making Life in an Introductory Linear Algebra Course Easier with Derive

In teaching linear algebra we have to deal with the following problem: while the level of the mathematical skills which are required to work with examples is generally low (students only need to add, subtract and multiply), the number of calculations is usually large. Therefore, working with examples is time-consuming and error-prone if done by hand. Students get tired quickly and lose their interest in this increasingly important area of mathematics. The faculty therefore decided to move the introductory linear algebra course from the classroom to the PC lab, and acquired a special DERIVE license that allows its use on all the PCs the faculty owns, and also on the private PCs of the students.

A utility file was then developed to facilitate teaching by providing functions for the computation of zero matrices and vectors, matrices and vectors of ones, as well as idempotent and orthogonal matrices, "just-in-time" whenever they are needed during the course. The utility file also contains functions that test if a given matrix is symmetric, idempotent or orthogonal.

4. Elimination Theory and Applications

org : Manfred Minimair, Seton Hall University, USA
Robert H. Lewis, Fordham University, USA

Room: B-2620

Schedule

Saturday June 27th

14:00 – 14:30	LU YANG, ZHENBING ZENG* AND WEINIAN ZHANG An Attempt to Apply Dixon Resultant to Differential Elimination
14:30 – 15:00	ROBERT H. LEWIS Solving Polynomial Systems of Autocatalytic Reactions in Chemistry
15:00 – 15:30	MANFRED MINIMAIR AND DEEPAK KAPUR Maximal-Rank Minors of the Macaulay Matrix

Sunday June 28th

8:30 – 9:00	CHANGBO CHEN, M. MORANO MAZA, BICAN XIA AND LU YANG Computing Cylindrical Algebraic Decomposition via Triangular Decomposition
9:00 – 9:30	DAN BATES Numerical determination of the local dimension of a solution of a polynomial system
9:30 – 10:00	B. PALANCZ, P. ZALETNYIK, J. AWANGE AND R.H. LEWIS Application of Symbolic - Numeric Algebra in Geodesy
10:30 – 11:00	GREGORY BARD Partitioning Multivariate Polynomial Equations via Vertex Cuts
11:00 – 11:30	SAMUEL LUNDQVIST Vector Space Bases Associated to Vanishing Ideals of Points
11:30 – 12:00	WEI PAN, MARC MORENO MAZA AND XIN LI *Computation modulo regular chains*

Abstracts

GREGORY BARD, Fordham University, USA
[Sunday, June 28th, 10:30, B-2620]
Partitioning Multivariate Polynomial Equations via Vertex Cuts

The variable-sharing graph of a polynomial system of equations has one vertex for each variable, and an edge between two variables if and only if those variables appear together in at least one equation. If this graph is disconnected, then the system is actually two separate systems that can be solved individually. This can provide a huge speed-up, but is unlikely to occur either randomly or in applications. However, it may be the case that deleting a small number of vertices c disconnects the variable-sharing graph in a balanced fashion, so that the ratio of the sizes of the larger and smaller components is roughly less than two.

If this is the case, then we demonstrate two techniques, one for small fields and one for large fields, to split the system. For small finite fields GF(q), we simply iterate through all possible q^c guesses of the c variables, and solve the separated systems. If the field is infinite or finite but large (i.e. has more than roughly five members), we separate the system with resultants. We present two methods for detecting this condition and identifying the c variables.

First, when c is small, or when {V} is small, one can run a large series of depth-first searches. Alternatively, when c or {V} is large, we show how to use off-the-shelf balanced edge cut determining software to generate a "reasonable" vertex cut. Also, we state a condition for a system of equations to be immune to this approach.

We present experiments which show that this condition can occur in practice, with very sparse polynomial systems. We also present four applications. The first is to implant/detect a trapdoor in the stream cipher QUAD; the second is to show a security property of the authentication system HB/HB+; the third has to do with "the Apollonius Problem" from Euclidean Geometry; the fourth is the "cube game" from multiparty game theory.

Dan Bates, Colorado State University, USA
[Sunday, June 28th, 9:00, B-2620]
Numerical determination of the local dimension of a solution of a polynomial system

A new symbolic-numerical local dimension test.
One of the sticky problems in numerical algebraic geometry over the years has been determining the local dimension of a solution p of a polynomial system F. The local dimension is just the maximum dimension of the irreducible components on which p sits. This new is a hybrid symbolic-numerical method that detects the dimension of _all_ components containing p by computing ranks of various matrices (formed from taking higher and higher partial derivatives of the system, more or less).

This is joint work with J. Hauenstein, C. Peterson, and A. Sommese.

Changbo Chen, Marc Moreno Maza, Becan Xia and Lu Yang, University of Western Ontario, Canada, Peking University and East China Normal University, China
[Sunday, June 28th, 8:30, B-2620]
Computing Cylindrical Algebraic Decomposition via Triangular Decomposition

Cylindrical algebraic decomposition is one of the most important tools for computing with semi-algebraic sets, while triangular decomposition is among the most important approaches for manipulating constructible sets. In this paper, for an arbitrary finite set F of polynomials with N variables with real coffecients, we apply comprehensive triangular decomposition in order to obtain an F-invariant cylindrical decomposition of the n-dimensional complex space, from which we extract an F-invariant cylindrical algebraic decomposition of the n-dimensional real space.

This new approach for constructing cylindrical algebraic decompositions has been implemented in the RegularChains library in Maple. We shall demonstrate its usage together with the other tools of this library for solving polynomial systems arising in real life problems.

Robert H. Lewis, Fordham University, USA
[Saturday, June 27th, 14:30, B-2620]
Solving Polynomial Systems of Autocatalytic Reactions in Chemistry

Autocatalytic reactions are chemical reactions in which at least one of the products is also a reactant. This "feed-back loop" yields a system of nonlinear polynomial equations. A famous example is called the Brusselator [1], [2]. Although the equations describing the classic Brusselator are trivial, the basic idea generalizes to more interesting situations [1], such as

$$a + x1^2*y1 - b*x1 - x1 + dx\,(x2 - x1),$$
$$b*x1 - x1^2*y1 + dy\,(y2 - y1),$$
$$a + x2^2*y2 - b*x2 - x2 + dx\,(x1 - x2),$$
$$b*x2 - x2^2*y2 + dy\,(y1 - y2)$$

We will examine the systems of equations that result from two- and three-dimensional configurations of interacting Brusselators. We have up to eight equations in eight variables and up to twelve parameters. We find that all are solvable with Dixon resultant methods. We will describe how various implementations of Groebner Bases fail on all but the simplest cases. We will show other examples of autocatalytic reactions.

[1] http://en.wikipedia.org/wiki/Autocatalytic_reaction

[2] Shaun Ault, http://fordham.academia.edu/ShaunAult/Papers/83373/Dynamics-of-the-Brusselator

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Samuel Lundqvist, Stockholm University, Sweden
[Sunday, June 28th, 11:00, B-2620]
Vector Space Bases Associated to Vanishing Ideals of Points

Let $k[x_1,\cdots,x_n]$ be the polynomial ring in n variables over a field *k*. The vanishing ideal with respect to a set of points in $\{p_1,\cdots,p_m\}$ in k^n is defined as the set of elements in $k[x_1,\cdots,x_n]$ that are zero on all of the p_i's.

The main tool that is used to compute vanishing ideals of points is the Buchberger-Möller algorithm, described in [1]. The Buchberger-Möller algorithm returns a Gröbner basis for the ideal vanishing on the set $\{p_1,\cdots,p_m\}$. A complementary result of the algorithm is a vector space basis for the quotient ring $k[x_1,\cdots,x_n]/I$. However, in many applications it turns out that it is the vector space basis, rather than the Gröbner basis of the ideal, which is of interest. For instance, it may be preferable to compute normal forms using vector space methods instead of Gröbner basis techniques.

A new bound for the arithmetic complexity of the Buchberger-Möller algorithm is given in [3], and is equal to $O(nm^2 + \min(m,n)m^3)$. We will discuss four constructions of vector space bases, all of which perform better than the Buchberger-Möller algorithm. An application of the constructions will be that we can drastically improve the method of the reverse engineering of gene regulatory networks given in [2].

The paper behind this work is accepted for publication in Journal of Pure and Applied Algebra. A preprint is available at http://arxiv.org/pdf/0808.3591/v2.

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MANFRED MINIMAIR AND DEEPAK KAPUR, Seton Hall University and University of New Mexico, USA
[Saturday, June 27th, 15:00, B-2620]
Maximal-Rank Minors of the Macaulay Matrix

We study under which conditions the maximal-rank minors of a (possibly singular) Macaulay matrix vanish. The Macaulay matrix is a matrix whose entries are the coefficients of a system of multivariate polynomial equations. The determinant of the Macaulay matrix vanishes if the polynomial system has a common root. Macaulay matrices have applications in many areas of computing, such as computer aided geometric design, robotics, computational chemistry, etc. It is shown that the vanishing of the maximal-rank minors of the Macaulay matrix of a parametric system of polynomials under specialization is a necessary condition for the specialized polynomials to have an additional common root even when the parametric system has common roots without any specialization of parameters. This result has applications where conditions for additional common roots of polynomial systems with generic roots are needed, such as in implicitization of surfaces with base points and in various other areas of computational geometry. We will discuss such applications.

B. PALANCZ, P. ZALETNYIK, J. AWANGE AND R.H. LEWIS, Budapest University of Technology and Economy, Hungary, Curtin University of Technology, Australia and Fordham University, USA
[Sunday, June 28th, 9:30, B-2620]
Application of Symbolic - Numeric Algebra in Geodesy

Most often geodetic problems are represented as system of multivariable polynomials. The bottleneck in solving such systems is finding out proper initial values for local iterative numerical methods. If the problem is relatively modest in size, computer algebraic methods like Groebner basis or Dixon resultant can give symbolic solution. To extend this result for overdetermined system one may use Gauss-Jacobi combinatoric solution employing the symbolic result many times. However, when the number of the combinatoric subsystems is very high, it is reasonable to employ global, robust numerical method like linear homotopy. In order to avoid the computation of all possible paths, consequently to simplify homotopy solution, the initial value of the start system can be evaluated from the symbolic solution of a subsystem. To illustrate this strategy a real world geodetic problem is presented. The 3 point problem of the 3D affine transformation is solved with Dixon resultant computed with EDF method. Then employing this result as initial value of the start system of linear homotopy as numerical method, the solution is computed for N = 1138 points as well.

WEI PAN, MARC MORENO MAZA AND XIN LI, University of Western Ontario and IBM Toronto, Canada
[Sunday, June 28th, 11:30, B-2620]
Computation modulo regular chains

The computation of triangular decompositions of polynomial systems is based on two fundamental operations: polynomial GCDs modulo regular chains and regularity test modulo saturated ideals.

We propose new algorithms for these core operations relying on modular methods and fast polynomial arithmetic. Our strategies take also advantage of the context in which these operations are performed.

We report on extensive experimentation, comparing our code to pre-existing Maple implementations, as well as more optimized Magma functions. In most cases, our new code outperforms the other packages by several orders of magnitude.

LU YANG, ZHENBING ZENG* AND WEINIAN ZHANG, East China Normal University, Chinese Academy of Sciences and Sichuan University, China
[Saturday, June 27th, 14:00, B-2620]
An Attempt to Apply Dixon Resultant to Differential Elimination

The idea of using resultant for solving differential equations can be traced to 1930's. The first work on "differential resultant" was done by O. Ore [19]. Later, the divisibility and elimination theory of algebraic differential equations was developed by C. H. Riquier [20], M. Janet [8], J. F. Ritt [21, 22], E. R. Kolchin [12] et al. The notion of resultant of two nonlinear differential

polynomials was introduced by Ritt [21] in 1932 under some hypothesis on the differential polynomials by using of pseudo-division. In 1997 G. Carra-Ferro [1] introduced a notion of differential resultant of two ordinary differential polynomials as quotient of two determinants and proved that a necessary condition for the existence of a common solution of two algebraic differential equations is that the differential resultant is equal to zero based on Macaulay's resultant (see [7, 17, 18]). Methods for using characteristic set theory to solve first order autonomous ODE and partial difference polynomial systems was propsed by X. S. Gao et al in [6]. Other generalizations of differential resultant can be found in [3], [11].

In this paper, we present our joint work on computing differential resultant via Dixon's resultant [10] for first and higher ODEs. Comparing with the classical resultant of Sylvester, the advantage of Dixon's resultant is that it can do one-step elimination of a block of unknowns from a system of polynomial equations like Macauley's. Meanwhile, there are works showing that Dixon's resultant is much faster than Macauley's for certain specific problems (cf. [13, 14, 15, 16]). So the motivation for using Dixon's resultant to solve differential resultants comes very natural. Our results in this paper show that this attempt is also worth to develop for many non-trivial problems listed in E. Kamke's book [9].

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5. Chemistry and Computer Algebra

org : Mihai Scarlete, Bishop's University, Sherbrooke, Canada
Manfred Minimair, Seton Hall University, USA
Robert H. Lewis, Fordham University, USA

Room: B-2624

Schedule

Saturday June 27th

8:30 – 9:00	MICHAEL ANTHONY WHITEHEAD Molecular Orbital Modelling of Dendrimers, and Nanotubes
9:00 – 9:30	R. H. LEWIS AND E. A. COUTSIAS Determining Flexibility of Molecules Using Resultants of Polynomial Systems
9:30 – 10:00	YEONG-NAN YEH Generalizations of Chung-Feller Theorems I
10:30 – 11:00	ZHENGHUA NIE Symbolic Construction of Spin Systems of the Liouville Space Method and its Applications
11:00 – 12:00	J. F. OGILVIE Fourier transforms in chemistry – diffraction and spectrometric applications
12:00 – 12:30	MIHAI SCARLETE, GAVIN HEVERLY-COULSON, STARR DOSTIE, AND KARL GAGNON Computer Algebra Application in Chemical Spectroscopy. Applications in Point Group Analysis of Molecular Vibration and Comprehensive Solving of Deceptively Simple Second Order Nuclear Magnetic Resonance
14:00 – 14:30	CASEY HENDERSON AND NECMETTIN YILDIRIM Computer Algebra Approach to Investigating Regulatory Mechanisms in Biochemical Reaction Networks: Application to Tryptophan Operon
14:30 – 15:00	SIMON J. FRASER On Enzyme Kinetics Using Manifold Theory

Abstracts

SIMON J. FRASER, University of Toronto, Canada
[Saturday, June 27th, 14:30, B-2624]
On Enzyme Kinetics Using Manifold Theory

Chemical reactions show a separation of time scales (rapid transient decay) due to the stiffness of the ordinary differential equations (ODEs) describing their evolution. In enzyme kinetics time scale separation allows the steady-state evolution of such systems to be represented on a hierarchy of smooth, slow manifolds embedded in the full phase space of concentration variables for the complete reaction. Typically such manifolds are dynamically stable in the sense that they attract the surrounding phase flow exponentially fast; this relates to their confinement within regions of phase space bounded by the nullclines of the system. The slow manifolds also contain the true attractors of the system. Explicit formulas for manifolds of this kind can be found by iterating functional equations using a symbolic language like Maple. It has been proved that, using sufficiently smooth starting functions, e.g., the expressions for the nullclines, the nth iteration of the functional equation provides expressions for the slow manifolds is accurate to the nth power in the singular perturbation parameter(s) that appear in the ODEs. However, the iteration procedure may diverge. This can be related to the geometry of the phase flow, e.g., the phase-space region in question does not lie between system nullclines and is not locally exponentially attracting. Nevertheless, if the local phase flow has the correct properties iteration can be stabilized.

The iterative method provides global formulas for the manifolds in cases where series methods diverge. There are many advantages to such reduced descriptions: the corresponding ODEs describe the system evolution on the slow manifolds; consequently, bifurcations of the system can be analysed on the manifolds: changes in the dimensionality associated with the system evolution can be expressed as structural, geometrical changes within the slow-manifold hierarchy.

CASEY HENDERSON AND NECMETTIN YILDIRIM, New College of Florida, USA

[Saturday, June 27th, 14:00, B-2624]

Computer Algebra Approach to Investigating Regulatory Mechanisms in Biochemical Reaction Networks: Application to Tryptophan Operon

Ordinary differential equations are widely used to study dynamics of biochemical reaction networks. Possible regulatory mechanisms in these networks can be investigated by exploring how the system variables (concentration) respond to small changes in parameter values. Mathematically, this can be done by solving the model and sensitivity equations simultaneously. The number of sensitivity equations is a product of the number of variables and parameters. Even for simple networks, the derivations of these equations becomes challenging, if not impossible, when one wants to do it by hand.

In this talk we will explain how to use Computer Algebra Systems, such as Maple, to perform a complete sensitivity analysis for biochemical reaction networks whose dynamics are described by a system of differential equations. In the second part of this talk, we will show how to apply our approach a model developed for the tryptophan operon of E.coli, which is a small genetic network responsible for production of tryptophan when it is not present in the environment.

R. H. LEWIS AND E. A. COUTSIAS, Fordham University and University of New Mexico, USA
[Saturday, June 27th, 9:00, B-2624]
Determining Flexibility of Molecules Using Resultants of Polynomial Systems

We solve systems of multivariate polynomial equations in order to understand flexibility of three dimensional objects, including molecules.

Protein flexibility is a major research topic in computational chemistry. In general, a polypeptide backbone can be modeled as a polygonal line whose edges and angles are fixed while some of the dihedral angles can vary freely. It is well known that a segment of backbone with fixed ends will be (generically) flexible if it includes more than six free torsions. Resultant methods have been applied successfully to this problem, see [3], [4].

In this work we focus on non-generically flexible structures (like a geodesic dome) that are rigid but become continuously movable under certain relations. The subject has a long history: Cauchy (1812), Bricard (1896), Connelly (1978).

In our previous work [8], we began a new approach to understanding flexibility, using not numeric but symbolic computation. We describe the geometry of the object with a set of multivariate polynomial equations, which we solve with resultants. Resultants were pioneered by Bezout, Sylvester, Dixon, and others. The resultant appears as a factor of the determinant of a matrix containing multivariate polynomials. Given the resultant, we described [8] an algorithm *Solve* that examines it and determines relations for the structure to be flexible. We discovered in this way the conditions of flexibility for an arrangement of quadrilaterals in Bricard [1], which models molecules. Here we significantly extend the algorithm and the molecular structures.

0.1 First new result

We have now analyzed Bricard's original formulation of the quadrilaterals problem [1] in terms of three quadratic equations, with fifteen parameters and three variables. The resultant of this system has 5685 terms. The flexibility searching algorithm is more subtle now, and we have modified algorithm *Solve* to include these cases, with great success. We have discovered an apparently new flexible arrangement, which can be viewed at [10]. Although the physically meaningful flexible conformations of the cyclohexane are well known ("chair" versus "boat"), this appears to be the first fully algebraic approach for their derivation, as well as for deriving Bricard's flexible octahedra. Moreover, the identical set of equations arises in other contexts, and a variant gives the conformational equations of a protein or nucleic acid backbone [3] [4].

0.2 Second new result

Next we consider the cylo-octane molecule, pictured in figure 1.

Chemically relevant solutions fix the (bond) angles between the paler lines, introducing four constraint equations in the variables τ_i. To save space, we show one equation here; the other three are similar.

$$-t\beta^4\,\tau_4{}^2\,\tau_1{}^2-4\,t\alpha_1\,t\beta^3\,\tau_4{}^2\,\tau_1{}^2+6\,t\beta^2\,\tau_4{}^2\,\tau_1{}^2+4\,t\alpha_1\,t\beta\,\tau_4{}^2\,\tau_1{}^2-\tau_4{}^2\,\tau_1{}^2-t\beta^4\,\tau_1{}^2+$$
$$4\,t\alpha_1{}^2\,t\beta^2\,\tau_1{}^2+2\,t\beta^2\,\tau_1{}^2-\tau_1{}^2-8\,t\alpha_1{}^2\,t\beta^2\,\tau_4\,\tau_1-8\,t\beta^2\,\tau_4\,\tau_1-t\beta^4\,\tau_4{}^2+4\,t\alpha_1{}^2\,t\beta^2\,\tau_4{}^2+$$
$$2\,t\beta^2\,\tau_4{}^2-\tau_4{}^2-t\beta^4+4\,t\alpha_1\,t\beta^3+6\,t\beta^2-4\,t\alpha_1\,t\beta-1=0$$

Here $\tau_i = \tan(z_i/2)$, $t\beta = \tan(\beta/2)$, and $t\alpha_i = \tan(\alpha_i/2)$.

We use the Dixon resultant to eliminate τ_2, τ_3, and τ_4. An important **special case** is when the basic quadrilateral (heavy black lines) is planar.

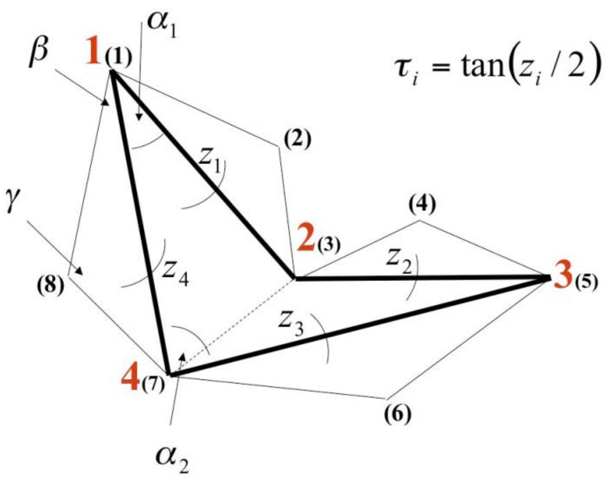

Fig. 1. Geometry of Octane Molecule.

The equations then simplify quite a bit, and we can describe all the solutions of this case. The Dixon matrix is 24×24. 57% of the entries are 0. On average there are 41 terms per entry. The Dixon-EDF method [9] takes 3 minutes 38 seconds to compute the resultant for τ_1, which has 21715 terms. It is degree 32 in τ_1 but has only even degree terms.

In the **general case** (three dimensional space) we have also made significant progress. The Dixon matrix is 64 × 64. 64% of the entries are 0. On average there are 107 terms per entry. The determinant of the Dixon matrix here, were it ever computed, would have many billions of terms. But our Dixon-EDF techniques [9] discover its hundreds of factors in about 67 hours of CPU time. The largest has 4872161 terms. Using some of these factors, we have verified some known chemical arrangements. We seem to have found new interesting flexible cases. Work is ongoing.

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ZHENGHUA NIE, McMaster University, Canada
[Saturday, June 27th, 10:30, B-2624]
Symbolic Construction of Spin Systems of the Liouville Space Method and its Applications

The first step to simulate the dynamics of spin systems is to construct the basic elements of Nuclear Magnetic Resonance (NMR) including the Liouvillian matrix, relaxation matrix, pulse matrix and equilibrium state. It is easy to deal with single spin systems, but there are more challenges to do coupled-spin systems. The computer algebra software provides the feasibility to build these elements for n-spin systems. Applying these elements, we can simulate and analyze the whole process of NMR experiments of more complex spin systems and pulse sequences in symbolic or numerical ways. In this talk, we will illustrate the computation of some experiments such as spin echo, steady state, INADEQUATE. We may also show how to efficiently compute the objective function, its first-order and second-order derivatives in the optimal pulse design.

J. F. OGILVIE, Simon Fraser University, Canada and Universidad de Costa Rica, Costa Rica
[Saturday, June 27th, 11:00, B-2624]
Fourier transforms in chemistry – diffraction and spectrometric applications

We employ Maple's ability to compute symbolic Fourier transforms of continuous curves to extract essential chemical information from interferograms produced with contemporary laboratory instruments. An interferogram is a function in the distance or time domain produced from the interference of waves, of which Fourier transformation into the wavenumber or frequency domain yields a distribution or spectrum that contains precise information about molecular structure and properties. We describe in turn each of four prototypical applications of continuous Fourier transforms – applied to diverse signals from coherent electron scattering, coherent xray scattering, microwave emission and infrared absorption – and then demonstrate how the capabilities of software for computer algebra enable the derivation of information in a chemically meaningful form. We distinguish between Fourier exponential, cosine and sine transforms that yield novel complementary information about molecular properties.

Mihai Scarlete, Gavin Heverly-Coulson, Starr Dostie, and Karl Gagnon, Bishop's University, Canada
[Saturday, June 27th, 12:00, B-2624]
Computer Algebra Application in Chemical Spectroscopy. Applications in Point Group Analysis of Molecular Vibration and Comprehensive Solving of Deceptively Simple Second Order Nuclear Magnetic Resonance

This contribution summarizes the advances in the utilization of computer algebra to the analysis of the activity of molecular vibration in Fourier-Transform Infrared Spectroscopy, and in the solving of complex Nuclear Magnetic Resonance spectra. The computation vehicle for the Point Group Analysis of molecular vibration is a Symbolic Computation Engine (SCE). In the first section, various computation modules are presented, designed for applications ranging from undergraduate teaching, to more elaborated versions integrated as important functionalities in advanced molecular modeling techniques used for research purposes. Fully integrated, digital character tables for most of point groups relevant for chemical species are built in SCE-compatible, comprehensive libraries. The advantages of this approach in teaching include manipulation of virtual molecules, step-by-step solutions for freshmen level, and fast, preformatted templates helping students to gain routine and eventually gain mastery on the subject. A special emphasis is put on the mathematical background of the calculations handled in SCE-code, covering the building of matrices associated to symmetry operators, computation of symmetry operators, coding of symmetry operations, point group analysis method, and matrix manipulations. In the second section, we introduce the utilization of computer-algebra in the reversible modeling-solving of deceptive AA'BB' NMR-spectra. The analysis of these spectra is hardly approachable by direct inspection or regular calculation. The continuous change of the spectrum to the limiting cases, and compatibility with the series of solutions bridging the deceptive cases to the known algorithms are discussed.

Michael Anthony Whitehead, McGill University, Canada
[Saturday, June 27th, 8:30, B-2624]
Molecular Orbital Modelling of Dendrimers, and Nanotubes

http://whitehead-group.mcgill.ca

Classical Mechanical Methods for calculating Molecular Structure and Quantum Mechanical methods for calculating Molecular Electronic Properties are reviewed. Appropriate Theoretical Techniques are used to investigate drendritic molecules containing silicon (Si) made from 3,5-

dihydroxybenzyl alcohol (DBHA), and tin (Sn) dendrimers made from 1,3,5 tri ethynyl benzene (TEB) as well as purely organic dendrimers of (TEB). Nanotubes formed from SMA (poly (styrene alt.maleic anhydride) [3] and from an imide derivative (SMI) [4] are considered. Finally the effect of water on the calcium carbonate CaCO3 Aragonite crystal (100) face is considered and contrasted with the effect of water on the (10.4) face of CaCO3 Calcite crystal.

YEONG-NAN YEH, Academia Sinica, Taiwan
[Saturday, June 27th, 9:30, B-2624]
Generalizations of Chung-Feller Theorems I

In this paper, we develop a method to find Chung-Feller extensions for three kinds of different root lattice paths and prove Chung-Feller theorems for such lattice paths. In particular, we compute a generating function $S(z)$ of a sequence formed by root lattice paths. We give combinatorial interpretations to the function of Chung-Feller type $\frac{S(z)-yS(yz)}{1-y}$ for the generating function $S(z)$. Using our method, we first prove Chung-Feller theorems of up-down type for three kinds of root lattice paths. Our results are generalizations of the classical Chung-Feller theorem of up-down type for Dyck paths. We also find Motzkin paths have Chung-Feller properties of up-down type. Then we prove Chung-Feller theorems of left-right type for two among three kinds of root lattice paths. Chung-Feller theorem of left-right type for Motzkin paths is a special case of our theorems. We also show that Dyck paths have Chung-Feller phenomenons of left-right type. By the main theorems in this paper, many new Chung-Feller theorems for root lattice paths are derived.

6. Applications of Math Software to Mathematical Research

org : Daniel Lichtblau, Wolfram Research, USA

Room: B-2620

Schedule

Friday June 26th

10:30 – 11:00	MHENNI M. BENGHORBAL Finding the nth Derivative and the nth Anti-Derivative Using Computer Algebra Systems
11:00 – 11:30	LUIS DAVID GARCIA-PUENTE Experimentation at the frontiers of reality in Schubert calculus
11:30 – 12:00	CHRISTOPH KOUTSCHAN Finding Algorithmic proving of special function identities in Mathematica
12:00 – 12:30	JIRI LEBL Uniqueness of certain polynomials constant on a hyperplane

Saturday June 27th

8:30 – 9:00	DANIEL LICHTBLAU Computing Knopfmacher's limit, or: My first foray into computational mathematics, reprise
9:00 – 9:30	ROBERT MCGRAIL Toward Computational Dichotomy for Finite Quandles
9:30 – 10:00	LUIS MEDINA Arithmetic properties of sequences satisfying first order recurrences
10:30 – 11:00	ALEKSANDR MYLLARI Conway Matrices Related to a Non-transitive Head-or-Tail Game with a q-sided die
11:00 – 11:30	TATIANA MYLLARI Modeling of the forest of random mapping with Mathematica
11:30 – 12:00	JAN VERSCHELDE Solving Schubert Problems with Littlewood-Richardson Homotopies
12:00 – 12:30	ERICH KALTOFEN Exact Certification in Global Polynomial Optimization Via Sums-Of-Squares of Rational Functions with Rational Coefficients

Abstracts

MHENNI M. BENGHORBAL, Concordia University, Canada
[Friday, June 26th, 10:30, B-2620]
Finding the nth Derivative and the nth Anti-Derivative Using Computer Algebra Systems

The aim of this work is to find closed form formulas that give the nth derivative and the nth anti-derivative of elementary and special functions. Here, we mainly concentrate on elementary functions and give some theorems and techniques for finding the nth derivative and the nth anti-derivative of integer orders. In general, n is a symbol, but it can be replaced by a real number. We will be focusing on the case when n is an integer.

The motivation of this work comes directly from the area of classical and fractional calculus as well as the area of symbolic computation. It is the answer to the question: Given a function f in a variable x, can computer algebra systems (CAS) find a formula for the nth derivative or the nth anti-derivative or both? A direct application of the nth derivative formulas is in the area of classical calculus. It is related to the construction of Taylor's series at a point x0 where one requires the nthn derivative of a function at the point where we approximate at. Other applications are related to solving ordinary and fractional differential equations.

In Maple, the formulas correspond to invoking the command diff(f(x), x$n) for differentiation. A software exhibition will be within the talk.

LUIS DAVID GARCIA-PUENTE, Sam Houston State University, USA
[Friday, June 26th, 11:00, B-2620]
Experimentation at the frontiers of reality in Schubert calculus

Joint work with Chris Hillar (MSRI), Abraham Martin del Campo (Texas A&M), James Ruffo (SUNY-Oneonta), Zach Teitler (Texas A&M), and Frank Sottile (Texas A&M).

In 1993 Boris and Michael Shapiro made a remarkable and (at the time) outlandish conjecture in the Schubert calculus, positing a way to ensure that all solutions to a given problem were real. This Shapiro conjecture became well-known due to large-scale computational experiments that supported it. This led to a partial solution by Eremenko and Gabrielov, and the full solution by Mukhin, Tarasov, and Varchenko in papers appearing in the Annals of Mathematics.

The Shapiro Conjecture concerns flags that are tangent to the rational normal curve. A proof of a special case of a generalization of it suggested that it may hold if tangent flags were replaced by secant flags. This Secant Conjecture has been the subject of a large-scale computational experiment that is not only seeking evidence for it, but also studying other, very subtle phenomena. Each instance is tested by generating a polynomial system that models a Schubert Calculus problem given by a choice of secant flags, and then determining the numbers of real and complex solutions.

In this talk, I will describe the setup, design, and running of this project. Largely using machines in instructional computer labs during off-hours and University breaks, this experiment has consumed over 250 GigaHertz-years of computation, testing over a billion instances of 274 different Schubert problems. Some involve as many as 42 solutions in a ten-dimensional space and require 9 GigaHertz-hours to compute a single instance. This experiment can serve as a model for other large-scale mathematical investigations.

ERICH KALTOFEN, North Carolina State University, USA
[Saturday, June 27th, 12:00, B-2620]
Exact Certification in Global Polynomial Optimization Via Sums-Of-Squares of Rational Functions with Rational Coefficients

We present a hybrid symbolic-numeric algorithm for certifying a polynomial or rational function with rational coefficients to be non-negative for all real values of the variables by computing a representation for it as a fraction of two polynomial sum-of-squares (SOS) with rational coefficients. Our new approach turns the earlier methods by Peyrl and Parrilo at SNC'07 and ours at ISSAC'08 both based on polynomial SOS, which do not always exist, into a universal algorithm for all inputs via Artin's theorem.

Furthermore, we scrutinize the all-important process of converting the numerical SOS numerators and denominators produced by block semidefinite programming into an exact rational identity. We improve on our own Newton iteration-based high precision refinement algorithm by compressing the initial Gram matrices and by deploying rational vector recovery aside from orthogonal projection. We successfully demonstrate our algorithm on 1. various exceptional SOS problems with necessary polynomial denominators from the literature, 2. very large (thousands of variables) SOS lower bound certificates for Rump's model problem (up to n = 17, factor degree = 16) and 3. for proving the Monotone Column Permanent Conjecture for dimension 4.

This is joint work with Bin Li, Zhengfeng Yang and Lihong Zhi.

CHRISTOPH KOUTSCHAN, Johannes Kepler University Linz, Austria
[Friday, June 26th, 11:30, B-2620]
Finding Algorithmic proving of special function identities in Mathematica

We present our newly developed Mathematica package HolonomicFunctions which serves for automatic finding and proving of holonomic function identities. The class of holonomic functions includes most of the elementary functions (like sine, exponential, logarithm, algebraic expressions, etc.) and the majority of so-called special functions (like Bessel functions, orthogonal polynomials, hypergeometric functions, etc.). The package adresses both summation and integration problems.

JIRI LEBL, University of Illinois, USA
[Friday, June 26th, 12:00, B-2620]
Uniqueness of certain polynomials constant on a hyperplane

Joint work with Daniel Lichtblau.

We study a question with connections to linear algebra, real algebraic geometry, combinatorics, and complex analysis. Let $p(x,y)$ be a polynomial of degree d with N positive coefficients and no negative coefficients, such that $p=1$ when $x+y=1$. It is known that the sharp estimate $d \leq 2N-3$ holds. We study the p that minimize N and we give complete classification of these polynomials up to $d=17$ by computational methods. We use separately a linear algebra approach and a mixed-integer programming approach. The question is motivated by a problem in CR geometry. In particular, a complete classification of polynomials minimizing N is an important first step in the complete classification of CR maps of spheres in different dimensions.

DANIEL LICHTBLAU, Wolfram Research, USA
[Saturday, June 27th, 8:30, B-2620]
Computing Knopfmacher's limit, or: My first foray into computational mathematics, reprise

I will discuss a problem I encountered over a decade ago, and worked on via internet with someone I (alas) never met. It involves a mix of number theory, real analysis, hard-core computation, and some slightly perplexing results.

In brief, we begin with a function expressed as a certain infinite product; Arnold Knopfmacher encountered it in an attempt to approximate the number of irreducible factors of univariate polynomials over Galois fields and raised the querstion of how to obtain a certain limit to this function. We derive and execute an effective algorithm for the task at hand. We'll also indicate why the most "obvious" approach does not work well in practice, or at all in theory.

ROBERT MCGRAIL, Bard College, USA
[Saturday, June 27th, 9:00, B-2620]
Toward Computational Dichotomy for Finite Quandles

Recent developments in universal algebra have revealed a significant relationship between the structure of finite algebras and the constraint satisfaction problem (CSP). The main goal of the Bard College Laboratory for Algebraic and Symbolic Computation (ASC) is to classify all finite quandles through the CSP. This talk presents an overview of this research project that focuses upon how systems such as GAP, KnotPlot, Mathematica, Prover9/Mace4, and SWI-Prolog have played essential roles.

LUIS MEDINA, Rutgers University, USA
[Saturday, June 27th, 9:30, B-2620]
Arithmetic properties of sequences satisfying first order recurrences

Let t_n be a sequence that satisfies a first order homogeneous recurrence $t_n = Q(n)t_{n-1}$, where $Q(n) \in \mathbb{Z}[n]$. These type of sequences arise in different types of problems like the integration of rational functions and the evaluation of infinite sums. In this talk, the asymptotic behavior of the p-adic valuation of t_n will be described.

ALEKSANDR MYLLARI, University of Turku, Finland
[Saturday, June 27th, 10:30, B-2620]
Conway Matrices Related to a Non-transitive Head-or-Tail Game with a q-sided die

We consider the well-known problem of string overlapping in connection with the so-called Penney Ante game with a q-sided die. Let two players, (1) and (2), agree on some integer $n \ge 2.$ Then both of them select a q-ary n-word, say w_1 and $w_2,$ and begin rolling a die until either w_1 or w_2 appears as a block of n consecutive outcomes. Player (1) wins if w_1 appears before w_2 does. The problem is to find the probability $P(w_1, w_2)$ that player (1) will win for the chosen w_1 and $w_2.$ A solution of the problem was proposed by J.H. Conway (but was not published), and the key tool of his solution is the so-called Conway matrix $C_n^{(q)}$ whose indices encode n consecutive outcomes.

We extend our previous results obtained for the case $q = 2$ ("coin") to the case of arbitrary $q >1.$ We propose a simple and effective algorithm for calculation and visualization of Conway matrices C_n and the corresponding matrices P_n (that give probabilities that player (1) will win for the chosen words w_1 and w_2) via the standard technique of the (inverse) multidimensional DFT. Computer Algebra system {\it Mathematica} (versions 5 and 6) was used for intermediate calculations, check of hypotheses and visualization of results.

This is joint work with N. Gogin and T. Myllari.

TATIANA MYLLARI, Abo Akademi University, Finland
[Saturday, June 27th, 11:00, B-2620]
Modeling of the forest of random mapping with Mathematica

We study random forests corresponding to random mappings. Mathematica 6 is used for modeling and visualization of results.

Let $T_n=\{1, 2, ..., n\}$ be a finite set with n elements. We represent a single-valued mapping S of a set T_n onto itself as

$$S = \begin{pmatrix} 1 & 2 & \dots & n \\ s_1 & s_2 & \dots & s_n \end{pmatrix}$$

where s_k denotes the element of the set T_n into which the element k is changed under the mapping S, k=1,2,...n. Mapping S can be represented as a directed graph with one arc leading from each vertex and joining the vertex to its image. The number of arcs incoming to the vertex is called the indegree of this vertex. Any connected component of a mapping graph contains exactly one cycle. After removing all arcs joining cycling vertices we get a forest of rooted trees, where cycling vertices are the trees roots and arcs are directed from vertices to the roots. We call this forest a mapping forest. The number of trees in this forest is equal to the number of cycling points of corresponding random mapping.

Let SIGMA_n be the set of all single-valued mappings T_n onto itself. We assume that the uniform distribution on the set SIGMA_n is given. It means that every mapping has the probability n^(-n). There are proved limit theorems for some characteristics of this set: the maximal size of tree in mapping forest, numbers of trees of the given size and the number of vertices with given indegree.

Our theoretical predictions for characteristics of forest of random mappings are checked by simulations, namely the number of vertices with given indegree. Random mapping of the set {1,2,,N}, where N is large enough number (100 000, 1 000 000, etc.), is generated. This mapping is further processed; its characteristics are calculated as well as characteristics of the corresponding forest: number of the cycles, number of the trees in the forest, etc. Number of the vertices with given degree is also calculated. These parameters are compared to the theoretically predicted values.

JAN VERSCHELDE, University of Illinois at Chicago, USA
[Saturday, June 27th, 11:30, B-2620]
Solving Schubert Problems with Littlewood-Richardson Homotopies

Given a sequence of nested linear spaces (called flags) and prescribed dimensions for each flag, a Schubert problem asks for all planes that meet the given flags at the prescribed dimensions. A geometric Littlewood-Richardson rule developed by Ravi Vakil leads to homotopy algorithms to solve a Schubert problem. Littlewood-Richardson homotopies are the families of polynomial systems constructed by these homotopy algorithms. Symbolically, homotopy algorithms degenerate a moving flag, using polynomial equations to keep conditions imposed by other flags fixed. At the degenerate configuration of the flag, a linear system provides a start solution for a path to track by numerical continuation methods. The specialization of a flag follows a combinatorial checker game. For sufficiently generic Schubert problems, the number of paths to track is optimal. The Littlewood-Richardson homotopies are implemented using the path trackers of the software package PHCpack.

This is work in progress joint with Frank Sottile and Ravi Vakil.

7. Computer Algebra for Dynamical Systems and Celestial Mechanics

org: Victor Edneral, Moscow State University, Russia
Aleksandr Myllari, University of Turku, Finland
Nikolay Vassiliev, Steklov Institute of Mathematics at St.Petersburg, Russia

Room: B-3432

Schedule

Sunday June 28th

8:30 – 9:00	Alexander Bruno and Victor Edneral On Integrability of a Planar ODE System
9:00 – 9:30	B.L. Markovski, O. Chuluunbaatar, A.A. Gusev and S.I. Vinitsky Symbolic algorithm for generating the multistep adiabatic representation
9:30 – 10:00	A. Mylläri, T. Mylläri , A. Rostovtsev and S. Vinitsky Formal Integral and Caustics in Dynamical Systems with Two Degrees of Freedom

Abstracts

Alexander Bruno and Victor Edneral, Keldysh Institute of Applied Mathematics of RAS and Skobeltsyn Institute of Nuclear Physics of Lomonosov Moscow State University, Russia
[Sunday, June 28th, 8:30, B-3432]
On Integrability of a Planar ODE System

We consider an autonomous system of ordinary differential equations, which is solved with respect to derivatives. To study local integrability of the system near a degenerate stationary point, we use an approach based on Power Geometry method and on the computation of the resonant normal form. For a planar 5-parametric example of such system, we found the complete set of necessary and sufficient conditions on parameters of the system for which the system is locally integrable near a degenerate stationary point.

B.L. Markovski, O. Chuluunbaatar, **A.A. Gusev** and **S.I. Vinitsky**, Joint Institute for Nuclear Research, Russia
[Sunday, June 28th, 9:00, B-3432]
Symbolic algorithm for generating the multistep adiabatic representation

e-mail: bmarkov2000@yahoo.com, vinitsky@theor.jinr.ru http://theor.jinr.ru/~chuka

Symbolic algorithm for generation of a step by step adiabatic approximation for the N-particle quantum mechanical problem is introduced in order to study the induced gauge fields is

developed. Application of the complex-vector-bundle formalism provides a natural framework for description of the multi-channel effective dynamics.

Some examples and practical realization of a first step of the above procedure, known in computational mathematics as the Kantorovich method [1] of reduction boundary problem for elliptic partial differential equations to systems of ordinary differential equations, are considered [2-4].

1. Kantorovich L. V. and Krylov V. I. Approximate Methods of Higher Analysis (NewYork, Wiley, 1964).
2. Chuluunbaatar O. et al. Comput. Phys. Commun.177, 649-675 (2007).
3. Chuluunbaatar O. et al. Comput. Phys. Commun.178, 301-330 (2008).
4. Chuluunbaatar O. et al. 2009 Comput. Phys. Commun. 10.1016/j.cpc.2009.04.017 (2009)

A. Mylläri, **T. Mylläri** , **A. Rostovtsev and S. Vinitsky**, University of Turku, Åbo Akademi University, Finland and Joint Institute for Nuclear Research, Russia
[Sunday, June 28th, 9:30, B-3432]

Formal Integral and Caustics in Dynamical Systems with Two Degrees of Freedom

We continue our studies of the feasibility of the usage of formal integral in investigating structure of caustics in dynamical systems with two degrees of freedom. H'enon-Heiles and Contopoulos models are used for experiments. We use the program LINA (version for Mathematica 6) developed at JINR to construct Gustavson-like formal integral of motion. This integral is used (together with the Hamiltonian of the system) to study analytically the evolution of caustics (changes in the structure of the velocity field) in the system. Results obtained analytically by using formal integral of motion are compared with the ones obtained by the numerical integration.

8. ANALOGY IN REASONING AND CONSTRUCTION

org: Michael P. Barnett, City University of New York, USA

Room: B-3420

Schedule

Saturday June 27th

8:30 – 8:35	Opening
8:35 – 9:10	MICHAEL BARNETT Using analogy in symbolic calculation
9:15 – 10:00	PIERRE BOUCHARD The Chemistry of Modern Combinatorics -- "Species' -- a Tool for Analogical Discussion
10:30 – 11:15	GODFRIED TOUSSAINT Analysis and Generation of Symbolically Represented Musical Rhythm
11:20 – 12:05	DEVIN GRIFFITHS The Semantic Analysis of Analogy in Darwin's Origin of Species
12:10 – 12:30	General discussion
14:00 – 14:45	PAUL THAGARD How brains compute analogies
14:50 – 15:30	Further discussion

Abstracts

MICHAEL BARNETT, City University of New York, USA
[Saturday, June 27th, 8:35, B-3420]
Using analogy in symbolic calculation

I have reduced the human effort in several recent symbolic calculations of chemical valence theory by the mechanized use of analogy. I explain the basic principles by (analogous) text processing examples, state the benefits of my mathematical applications of these methods, and show some diagrammatic depictions of certain laboratory experiments constructed in analogous ways.

PIERRE BOUCHARD, Université of Québec à Montréal, Canada
[Saturday, June 27th, 9:15, B-3420]
The Chemistry of Modern Combinatorics -- "Species' -- a Tool for Analogical Discussion

Category theory is a flourishing branch of mathematics that has the exploration of analogy as one of its major uses. I will give a very brief and naive introduction to the concepts of "category" and "functor". Then I will give simple definitions of a "species of structure F" and a "structure of species F over a finite set U", illustrated by numerous examples. I will then show what a "molecular" species is, give all examples of molecular species on a small number of elements, and use an example to show how every species can be decomposed into a "sum" of molecular species. From there I will show how to count the number of molecular species that there can be on a given finite set of n elements, and what two chemists found about that in 1929. To conclude, I will mention what an atomic species is, and show how any molecular species can be decomposed into a "product" of atomic species. Where can we go from there? The answer during the talk!

DEVIN GRIFFITHS, Rutgers University, USA
[Saturday, June 27th, 11:20, B-3420]
The Semantic Analysis of Analogy in Darwin's Origin of Species

I discuss the role of analogy in Darwin's "Origin of Species" using textual analysis. One of his key contributions was to redefine "evolution", "species" and other basic concepts. I analyze single-sentence analogical constructions over the course of his work using Latent Semantic Analysis (LSA) in novel ways that include Fisher testing, leading to a discussion of analogy and its relationship to the coherence of sentence-to-sentence meaning. Analogies transfer characteristics between semantic systems and I reflect upon what this implies.

PAUL THAGARD, University of Waterloo, Canada
[Saturday, June 27th, 14:00, B-3420]
How brains compute analogies

In our 1996 book, Mental Leaps, Keith Holyoak and I presented a psychological theory of how people perform analogical thinking. We argued that retrieval, mapping, and transfer of analogical information involve 3 constraints: similarity, structure, and purpose. In the past decade, much has been learned about the neural processes that underlie these constraints. This talk will offer a computational account of how neural networks compute similarity, structure, and purpose.

GODFRIED TOUSSAINT, McGill University, Canada
[Saturday, June 27th, 10:30, B-3420]
Analysis and Generation of Symbolically Represented Musical Rhythm

Tools for analyzing musical rhythms that are represented symbolically are reviewed. Such tools include measuring the complexity and similarity of rhythms, as well as characterizing what makes a "good" rhythm good. These characterizations lead to simple algorithms for automatic generation of "good" rhythms. Analogy plays an important role throughout this work.

9. Symbolic and Numeric Computation

org: Alkis Akritas, University of Volos, Greece
Hiroshi Kai, Ehime Univ., Japan
Itnuit Janovitz-Freireich, CINVESTAV-IPN, Mexico
Doru Stefanescu, University of Bucharest, Romania

Room: B-3432

Schedule

Saturday June 27th

8:30 – 9:00	HIROSHI KAI A MathML authoring tool using approximate algebra
9:00 – 9:30	DORU STEFANESCU Improved methods for computation of bounds for positive polynomial roots
9:30 – 10:00	HIROSHI MURAKAM Solution of Approximated Roots of Equation by the Orthogonal Polynomial Expansion
10:30 – 11:00	ITNUIT JANOVITZ-FREIREICH On the Computation of Matrices of Traces and Radicals of Ideals
11:00 – 11:30	ADRIEN POTEAUX Computing Puiseux espansions : a new symbolic-numeric algorithm
11:30 – 12:00	DAN BATES Exact ideals from numerical data
14:00 – 14:30	YUN GUAN Conditioning numerical representations of algebraic sets
14:30 – 15:00	GUILLAUME MOROZ Chebyshev expansion for yet another paving algorithm

Abstracts

DAN BATES, Colorado State University, USA
[Saturday, June 27th, 11:30, B-3432]
Exact ideals from numerical data

Numerical homotopy methods, pioneered largely by Sommese, Verschelde, and Wampler, may be used to construct a numerical irreducible decomposition of an algebraic set. In particular, these methods will produce at least one approximation of a generic point on each irreducible component of the solution set of a set of polynomials. Applying a method such as LLL or PSLQ to certain embeddings of one of these approximate generic points will yield all integer relations (exact defining equations) for the irreducible component on which the point lies. As a result, one may recover at least some of the information contained in the symbolic decomposition of an ideal without relying on symbolic methods such as Groebner basis computations. This is ongoing work with J. Hauenstein, T. McCoy, C. Peterson, and A. Sommese.

YUN GUAN, University of Illinois at Chicago, USA
[Saturday, June 27th, 14:00, B-3432]
Conditioning numerical representations of algebraic sets

Numerical data structures for positive dimensional solution sets of polynomial system are sets of generic points cut out by random planes. We may represent the linear spaces defined by those planes either by explicit linear equations or in parametric form. These descriptions are respectively called extrinsic and intrinsic representations. Previous work by Andrew Sommese, Jan Verschelde, and Charles Wampler showed how the intrinsic formulation of diagonal homotopies reduced the cost of the linear algebra operations during path following. Via coordinate transformations we observed improved conditioning and an increased radius of convergence of Newton's method. This is work in progress joint with Jan Verschelde.

ITNUIT JANOVITZ-FREIREICH, Centro de Investigación y Estudios Avanzados del I.P.N, Mexico
[Saturday, June 27th, 10:30, B-3432]
On the Computation of Matrices of Traces and Radicals of Ideals

Consider a system of polynomials generating a zero-dimensional ideal I. We study the computation of matrices of traces for the factor algebra A, i.e. matrices with entries which are trace functions of the roots of I. Such matrices of traces in turn allow us to compute a system of multiplication matrices of the radical I. We first propose a method using Macaulay type resultant matrices to compute moment matrices, and in particular matrices of traces for A. We prove bounds on the degrees needed for the Macaulay matrix in the case when I has finitely many projective roots. We also extend previous results which work only for the case where A is Gorenstein to the non-Gorenstein case. The second proposed method uses Bezoutian matrices to compute matrices of traces of A. This second method also works if we have higher dimensional components at infinity. A new explicit description of the generators of the radical are given in terms of Bezoutians. This work was done in collaboration with Bernard Mourrain, Lajos Ronyai and Agnes Szanto.

HIROSHI KAI, Ehime University, Japan
[Saturday, June 27th, 8:30, B-3432]
A MathML authoring tool using approximate algebra

We present an implementation of a MathML authoring tool on the xfy. The xfy is a software developed in Justsystems Co., that aims at creating an authoring environment for compound XML documents in a single workspace. In the context authoring MathML on the xfy, we may utilize computer algebra systems to input or plot expressions. When the inputs are inexact polynomials, we will show approximate algebra, such as approximate-GCD, can be applied to author expressions or draw a curve with precision ϵ. This feature of the MathML editor can be useful to author any scientific documents or educational contents for learning management systems (e.g. Moodle). In this talk, its implementation and reliability will be discussed. Authors : Masaaki Kataoka (Ehime University, Japan), Hironori Shimazu (Ehime University, Japan), Hiroshi Kai (Ehime University, Japan), Atsushi Miyamoto (Justsystems Co., Japan)

GUILLAUME MOROZ, Maplesoft, Canada
[Saturday, June 27th, 14:30, B-3432]
Chebyshev expansion for yet another paving algorithm

The problem of paving the solution of polynomial inequalities with rectangular boxes has many applications in real geometry: among the most important ones are volume computation of semi-algebraic sets, and uniform point sampling. Consider the inequality p>0, where p is a multivariate polynomial. The problem of box paving consists in approximating the set of solutions of p>0 with rectangular boxes. In the framework of spatial subdivision tree algorithms, the main issues are: 1. the design of a sufficient criterion to detect if p has no real zero in a given box 2. the design of heuristics to split a box when the criterion doesn't allow us to conclude Many approaches have been used to address these issues, including Taylor expansion, Bernstein expansion, adapted narrowing operators... On another hand, while the Chebyshev polynomials have very good properties for numerical approximations of functions, they have rarely been used in the context of multi-dimensional box paving. Indeed, we will show that the Chebyshev expansion has very nice properties for the problem of box paving, and we will present the results we obtained with a high-level implementation in maple.

HIROSHI MURAKAM, Tokyo Metropolitan University, Tokyo, Japan
[Saturday, June 27th, 9:30, B-3432]
Solution of Approximated Roots of Equation by the Orthogonal Polynomial Expansion

When the roots are searched in the interval, the algebraic equation is expanded by the orthogonal polynomials associated with the interval. The roots are solved in the neighbor of the interval as the eigenvalues of the generalized companion matrix which corresponds to the orthogonal expansion. Similarly, the smooth non-linear equation is approximated by the truncated orthogonal expansion in the interval, and the approximated roots of the non-linear equation in the interval are solved by the above generalized companion method as the roots of the expansion. The condition of the set of orthogonal polynomials in the interval is much better than that of monomials (x-c)^k (which is also the basis for Taylor expansion). Therefore, the calculated approximated roots are expected to give smaller residuals with the restricted precision of the floating point numbers. Some examples would be shown which solved equations approximately by the applications of the orthogonal polynomial expansion.

Adrien Poteaux, University of Western Ontario, Canada
[Saturday, June 27th, 11:00, B-3432]
Computing Puiseux espansions : a new symbolic-numeric algorithm

Given a bivariate polynomial $F \in k[x,y]$ and α a root of the discriminant of F in y, computing nunmerical approximations of Puiseux series of F above α (i.e. the series in $(x-\alpha)$ solutions of the polynomial F viewed as a univariate polynomial in y) is not an easy task. Usual algorithms, namely the Newton-Puiseux algorithm and its variants, are symbolic algorithms. Computing Puiseux series symbolically may be costly, because of the extension fields involved and the coefficients growth. Moreover, a numerical evaluation of these coefficients may sometimes need a high number of digits due to devastating cancellations. On the other hand, pure numerical computations cannot be used directly: the slightest approximation causes Newton-Puiseux algorithm to miss essential information, such as ramification indices. It also causes numerical instabilities, since any close approximation of α_0 of α leads to expansions with a very small convergence radius $alpha-\alpha_0|$. To resolve the matter, we describe a new symbolic-numeric strategy. Indeed, studying the Newton-Puiseux algorithm, we can note that only two type of informations are needed exactly: Newton polygons and multiplicity structures of the characteristic polynomials. Thus, our strategy can be resume as follows: we first compute the Puiseux series modulo a well chosen prime number p. This give us the structure of the Puiseux series, namely the "polygon tree". Then, we show how to follow this polygon tree to compute numerical approximations of the Puiseux series coefficients. This is a work made during my PhD at the University of Limoges, in collaboration with Marc Rybowicz.

Doru Stefanescu, University of Bucharest, Romania
[Saturday, June 27th, 9:00, B-3432]
Improved methods for computation of bounds for positive polynomial roots

We discuss applications of some new results on univariate polynomials with real coefficients to the computation of bounds for positive roots. The computation of such bounds is a key step in CF_algorithms for real root isolation. We study the optimality and discuss strategies for improving the computation of upper and lower bounds. Our results are compared with classical methods and with results of Kioustelidis, Stefanescu and Akritas-Strzebonski-Vigklas.

10. Algebraic and Algorithmic Aspects of Differential and Integral Operators

org: Markus Rosenkranz, Austrian Academy of Sciences, RICAM, Austria
Georg Regensburger, Austrian Academy of Sciences, RICAM, Austria

Room: B-2620

Schedule

Thursday June 25th

10:30 – 11:00	Moulay Barkatou Local Reduced Forms of Systems of Linear Functional Equations and Applications
11:00 – 11:30	Thomas Cluzeau Serre's Reduction of Linear Functional Systems: Theory, Implementation and Applications
11:30 – 12:00	Li Guo, William Sit and Ronghua Zhang Rota-Baxter Type and Differential Type Algebras
12:00 – 12:30	Christoph Koutschan A Difference Operators Attack on Hard Combinatorial Problems
14:00 – 14:30	George Labahn Normal Forms of Matrices of Differential Polynomials
14:30 – 15:00	Xuan Liu and Greg Reid Symmetry Operators and Differential Equations
15:00 – 15:30	Leon Pritchard and William Sit Algebraic Constraints on Initial Values of Differential Equations
16:00 – 16:30	Alban Quadrat A Normal Form for 2-dimensional Linear Functional Systems
16:30 – 17:00	Georg Regensburger, Markus Rosenkranz and Johannes Middeke Integro-Differential Operators as an Ore Algebra
17:00 – 17:30	Kate Shemyakova and Elizabeth Mansfield Moving Frames for Laplace Invariants. General Case

Friday June 26th

8:30 – 9:00	Hynek Baran Classifying integrable systems via the spectral parameter problem
9:00 – 9:30	Hiroshi Umemura Discrete Burgers' Equation, Binomial Coefficients and Mandala
9:30 – 10:00	Yang Zhang Irreducibility Criteria for Skew Polynomials

Abstracts

HYNEK BARAN, Silesian University in Opava, Czech Republic
[Friday, June 26th, 8:30, B-2620]
Classifying integrable systems via the spectral parameter problem

We consider the problem of systematic classification of integrable PDE initially known to possess a nonparametric zero curvature representation. The method is taken from M. Marvan, On the spectral parameter problem, online on Acta Appl. Math., DOI 10.1007/s10440-009-9450-4 or http://arxiv.org/abs/0804.2031v2 . Discussed will be an implementation of the method and results of classification of integrable Weingarten surfaces in E^3.

MOULAY BARKATOU, Université de Limoges, France
[Thursday, June 25th, 10:30, B-2620]
Local Reduced Forms of Systems of Linear Functional Equations and Applications

Moser and super-irreducible forms play a central role in the local analysis (and hence in the global study) of liner systems of differential or difference equations. Algorithms for constructing such forms have been developed and implemented in the Maple package ISOLDE. In this talk, we shall explain how these concepts can be extended to the general class of *systems of pseudo-linear equations* which comprises common types of systems such as linear differential, difference or *q*-difference systems. For this we first introduce a unifying framework that permits us to treat, simultaneously, all types of linear functional systems. This is done by using the language of pseudo-linear derivations over a field of discrete valuation. We derive a definition of regularity and develop a method for recognizing regular systems inspired by Moser's work on differential equations.

As an application we shall show that the approach we developed in the past for computing formal series solutions for differential and difference systems is also applicable for *q*-difference systems.

This talk is based on a recent joint work with E. Puegel and G. Broughton.

THOMAS CLUZEAU, Université de Limoges, France
[Thursday, June 25th, 11:00, B-2620]
Serre's Reduction of Linear Functional Systems: Theory, Implementation and Applications

The first purpose of this talk is to recall briefly constructive results on Serre's reduction of determined / overdetermined / underdetermined linear functional systems obtained recently by M. S. Boudellioua (Sultan Qaboos University, Oman) and A. Quadrat (INRIA Sophia Antipolis, France). Serre's reduction aims at finding an equivalent presentation of a linear functional system which contains fewer equations and unknowns. We shall explain why this problem can be reduced to the case where the equivalent system contains only one equation. Then, we will discuss our implementation of these results in an OreModules package called Serre. Finally, we will concentrate on the zero-dimensional case (D-finite or holonomic linear functional systems) where we can go further in this analysis and explain the links between these results and the Jacobson normal form of a matrix with entries in a principal ideal domain (e.g., ODEs with rational coefficients). The different results will be illustrated with explicit examples coming from control theory and engineering sciences. This is a work in progress in collaboration with A. Quadrat (INRIA Sophia Antipolis, France).

LI GUO, WILLIAM SIT AND RONGHUA ZHANG, Rutgers University and City University of New York, USA
[Thursday, June 25th, 11:30, B-2620]
Rota-Baxter Type and Differential Type Algebras

A Rota-Baxter *k*-algebra *R* is a (not necessarily commutative, but associative) *k*-algebra (where *k* is a unitary commutative ring) with a linear operator $P : R \to R$ satisfying the following Rota-Baxter identity for all *x*, $y \in R$:

$$P(x)P(y) = P\big(P(x)y + xP(y)\big)$$

The identity reflects the integration-by-parts formula of calculus, but there are many other examples of Rota-Baxter algebras. A differential *k*-algebra *R* is (usually commutative and associative) *k*-algebra with a linear operator $\delta : R \to R$ satisfying the Leibniz identity for all *x*, $y \in R$:

$$\delta(xy) = \delta(x)y + x\delta(y).$$

In this talk we describe a framework to solve a still open problem of Rota: to classify all possible Rota-Baxter type algebras, where the Rota-Baxter identity is modified or generalized. Dually, the framework will also work for differential type algebras when the Leibniz identity is modified or generalized. We produce through computation 15 classes of Rota-Baxter type algebras, many of which are new. Research in the case for differential type algebras has not even begun, although some work on differential Rota-Baxter algebras has started.

Both Rota-Baxter algebras and differential algebras have wide applications such as in combinatorics, number theory, quantum field-theory and differential equations.

CHRISTOPH KOUTSCHAN, Johannes Kepler University, Austria
[Thursday, June 25th, 12:00, B-2620]
A Difference Operators Attack on Hard Combinatorial Problems

We use the algebra of difference operators (which we introduce as a discrete analogue of differential operators) in order to prove the enumeration formula for totally symmetric plane partitions (TSPP), whose proof by hand is highly nontrivial (Stembridge 1995). Our computer algebra (holonomic systems) approach is a computational challenge that became feasible only by using new techniques. The remarkable point in our new proof of TSPP is that the same technique can be applied to the generalized problem q-TSPP which is still an outstanding open problem in enumerative combinatorics.

GEORGE LABAHN, University of Waterloo, Canada
[Thursday, June 25th, 14:00, B-2620]
Normal Forms of Matrices of Differential Polynomials

Normal forms for matrix polynomials (such as Hermite, Popov etc) have been very useful in many areas of mathematics (e.g. linear control theory). In this talk we will discuss normal forms for matrices of differential operators. We show their usefulness in the context of systems of linear differential equations and discuss the various computational challenges in computing the forms for arbitrary matrices of differential operators.

XUAN LIU AND **GREG REID**, University of Western Ontario, Canada
[Thursday, June 25th, 14:30, B-2620]
Symmetry Operators and Differential Equations

Finite and infinite Lie symmetry pseudo-groups of differential equations are sets of continuous transformations leaving invariant their solution spaces. They are central to the modern exact approaches for nonlinear differential equations such as reduction of dimension, determining invariant solutions and mappings. In this talk we give an introduction to such methods and the algorithmic determination of the structure of such pseudo-groups; and the related algebras of operators.

LEON PRITCHARD AND **WILLIAM SIT**, City University of New York, USA
[Thursday, June 25th, 15:00, B-2620]
Algebraic Constraints on Initial Values of Differential Equations

Initial value problems of differential algebraic equations are of practical importance in many applications. Three related problems have been investigated: singularities, numerical solutions, and existence and uniqueness of solutions. We will briefly discuss the difficulties posed by each. In this talk, we describe a computational approach to obtain algebraic constraints on initial values that would guarantee existence and uniqueness of solutions. These constraints may be implicitly implied by the differential equations themselves. We apply this approach to a class of non-linear systems of first order ordinary differential equations and in addition to obtaining the constraints, the algorithms will also provide equivalent systems where the first derivatives of the dependent variables (unknowns) are explicitly given in terms of the unknowns. This vector field can be integrated in a numerically stable way. Examples where singularities are exposed by the algorithm will be given.

ALBAN QUADRAT, Institut National de Recherche en Informatique et en Automatique, France
[Thursday, June 25th, 16:00, B-2620]
A Normal Form for 2-dimensional Linear Functional Systems

Jacobson normal forms of matrices with entries in polynomial rings of ordinary differential or difference operators with coefficients in a skew field play an important role in the study of linear systems of ordinary differential or recurrence equations. Unfortunately, they generally do not exist for matrices with entries in noncommutative polynomial rings in more than one variable, i.e., they cannot be used to study linear systems of partial differential equations, differential time-delay systems or multi-indexed recurrence equations.

The purpose of this talk is to show that every matrix over a noncommutative polynomial ring in two independent variables admits an upper triangular reduction formed by three diagonal blocks: the first diagonal block defines the torsion-free part of the linear system, the second one defines the 1-dimensional part and the third one defines the 0-dimensional part of the system. Hence, the corresponding linear system can be integrated in cascade by first solving the 0-dimensional system, then the 1-dimensional one and finally the parametrizable one. This form for 2-dimensional linear systems generalizes the Jacobson normal form for 1-dimensional linear systems. This normal form, based on difficult results of module theory (e.g., pure submodules, purity filtration) and homological algebra (e.g., extension functor, ext_D^i(ext_D^i(M, D), D), Baer's extensions), can be computed by means of Gröbner or Janet basis techniques.

Georg Regensburger, **Markus Rosenkranz** and **Johannes Middeke**, Austrian Academy of Sciences and Johannes Kepler University, Austria
[Thursday, June 25th, 16:30, B-2620]
Integro-Differential Operators as an Ore Algebra

The notion of integro-differential algebra brings together the usual derivation structure with an algebraic version of indefinite integration and evaluation. We construct the associated algebra of integro-differential operators (used for modeling Green's operators for linear boundary problems) directly in terms of normal forms. For polynomial coefficients, we can use skew polynomials, defining the integro-differential Weyl algebra as a natural extension of the classical Weyl algebra in one variable. Its algebraic properties and its relation to the localization of differential operators are studied. Fixing the integration constant, we regain the integro-differential operators with polynomial coefficients.

Kate Shemyakova and **Elizabeth Mansfield**, Johannes Kepler University, Austria, and University of Kent, UK
[Thursday, June 25th, 17:00, B-2620]
Moving Frames for Laplace Invariants. General Case

Using the methods of regularized moving frames of Fels and Olver we obtained in our ISSAC'08 paper generating sets of invariants for the second- and third-orders bivariate hyperbolic and non-hyperbolic LPDOs. Now we generalize the method to the case of arbitrary order (hyperbolic and non-hyperbolic) LPDOs. A special choice of the normalization equations was invented, so that we were able prove some invariants form a finite (small) generating set of invariants.

HIROSHI UMEMURA, Nagoya University, Japan
[Friday, June 26th, 9:00, B-2620]
Discrete Burgers' Equation, Binomial Coefficients and Mandala

Burgers' equation is a fundamental equation in fluid mechanics. We know its discretization and ultra-discretization. The ultra-discretization describes traffic flow. We talk about the discretization but over the prime field $F_2=Z/2Z$. The equation generates a beautiful pattern, which we call mandala.

YANG ZHANG, University of Manitoba, Canada
[Friday, June 26th, 9:30, B-2620]
Irreducibility Criteria for Skew Polynomials

In this talk, we present two irreducibility criteria for the elements of a large class of skew-polynomial rings. The proofs rely heavily on non-commutative valuations and extensions thereof. The results apply, in particular, to ordinary linear differential operators and linear difference operators having coefficients in not-necessarily-commutative fields with valuations. This is a joint with R. Churchill.

11. HIGH-PERFORMANCE COMPUTER ALGEBRA

org: Jeremy Johnson, Drexel University, USA
Marc Moreno Maza, University of Western Ontario, Canada

Room: B-2624

Schedule

Thursday June 25th

16:00 – 16:30	JAMES DAVENPORT A computer algebraist meets a computer centre director
16:30 – 17:00	BENOIT LACELLE AND ERIC SCHOST Towards an efficient implementation for the resolution of structured linear system
17:00 – 17:30	ERIC ROBINSON, GENE COOPERMAN, DANIEL KUNKLE AND JÜRGEN MÜLLER Parallel Disk-Based Computation and Computational Group Theory
17:30 – 18:00	JAN VERSCHELDE Multitasking Polynomial Homotopy Continuation in PHCpack

Friday June 26th

8:30 – 9:00	DANIEL ROCHE Memory Efficiency in Polynomial Multiplication
9:00 – 9:30	LING DING AND ERIC SCHOST – Presented by MARC MORENO MAZA Fast multiplication and its variants in Newton iteration
9:30 – 10:00	YUZHEN XIE AND MARC MORENO MAZA Balanced Dense Polynomial Multiplication on Multi-cores
10:30 – 11:00	JEREMY JOHNSON AND LINGCHUAN MENG SPIRAL-Generated Modular FFTs
11:00 – 11:30	JEAN-GUILLAUME DUMAS, DAVID SAUNDERS* AND BRYAN YOUSE Linear Algebra Modulo Tiny Primes
11:30 – 12:00	WEI PAN, XIN LI AND MARC MORENO MAZA Implementing Modular Methods in Maple with the Modpn library
12:00 – 12:30	JEREMY JOHNSON, WERNER KRANDICK, DAVID RICHARDSON AND ANATOLE RUSLANOV Code Generation and Autotuning in Computer Algebra

Abstracts

JAMES DAVENPORT, University of Bath, UK
[Thursday, June 25th, 16:00, B-2624]
A computer algebraist meets a computer centre director

Most supercomputers are designed for, and bought for, floating-point computations. Even here, it can be surprisingly difficult to get the advertised performance, as the author, a (super-)computer centre director observed. In particular, a great deal of performance depends on the regularity of the data, and exploiting the parallelism available within an individual core, between cores on anode, and between nodes. So how does the author, a computer algebraist, convince the director that he can make effective use of the supercomputer?

LING DING AND ERIC SCHOST, University of Western Ontario, Canada
[Friday, June 26th, 9:00, B-2624]
Fast multiplication and its variants in Newton iteration

We discuss various forms of Newton iteration, for computing power series solutions of differential or polynomial equations. We show how to apply fast multiplication techniques such as short product or middle product in a systematic manner, generalizing some previous approaches known for e.g. power series inverse or square root.

JEAN-GUILLAUME DUMAS, DAVID SAUNDERS* AND BRYAN YOUSE, Université Joseph Fourier, France and University of Delaware, USA
[Friday, June 26th, 11:00, B-2624]
Linear Algebra Modulo Tiny Primes

Numerous applications require linear algebra modulo 2, 3, 5 or another very small prime. For example in the study of graphs, p-rank of adjacency matrices is often a useful discriminator. Integer Smith Normal Form computation by modular methods usually has as its greatest challenge the determination of the tiny prime contribution to the invariant factors.

Modular residues may be packed into machine words so that several arithmetic computations are done in one machine instruction. We report on several packing schemes including packing into doubles and using floating point arithmetic, packing into ints and using a combination of int arithmetic and bit operations, finally a scheme called bit-slicing.

We report performance measurements of the packing strategies on several architectures and discuss tuning issues. Finally we discuss the software design concerning the incorporation of such packing in the LinBox C++ template library for exact linear algebra. The question is how to factor the code so that packed and unpacked vectors are equally usable by algorithms that are generic with respect to ground field representation, vector details, and matrix representation.

JEREMY JOHNSON, WERNER KRANDICK, DAVID RICHARDSON AND ANATOLE RUSLANOV, Drexel University and SUNY Fredonia, USA
[Friday, June 26th, 12:00, B-2624]

Code Generation and Autotuning in Computer Algebra

Recently many numeric computing kernels such as matrix multiplication [ATLAS and PHiPAC], sparse matrix vector product [Sparsity] the fast Fourier transform [FFTW, UHFFT], and more general DSP transforms [SPIRAL] have utilized code generation and automated performance tuning to obtain high-performance implementations tuned to specific platforms. This talk outlines how these techniques may be applied to symbolic computation and computer algebra systems.

Examples demonstrating the benefit of these techniques are drawn from the papers [ISSAC06, ISSAC05] which show that the performance of the Taylor shift operation used in real root isolation can be substantially improved through delayed carry propagation, radix reduction, and an interlaced coefficient representation combined with register tiling.

This speedup is obtained over a straightforward implementation that utilizes the heavily optimized integer addition routines from GMP. The resulting code is significantly more complicated that the original code utilizing GMP. Moreover, the code can be tuned using different tile sizes, amounts of unrolling and different scheduling strategies. Note that compilers are not capable of performing this tuning due to various dependencies and lack higher knowledge needed to transform the code.

The particular tuning parameters used depends on the platform (e.g. number of registers, number of integer units, etc.) and furthermore, the optimal parameter settings are not easily determined without runtime experiments. This talk presents a code generator from [Richardson09] that produces code with different tile sizes and empirically searches for the best implementation on a given platform. [ISSAC06] Jeremy Johnson, Werner Krandick, Kevin Lynch, David Richardson, and

Anatole Ruslanov, High-performance implementations of the Descartes method. In J.-G. Dumas, editor, International Symposium on Symbolic and Algebraic Computation, pp. 154-161, ACM Press, 2006.

[ISSAC05] Jeremy Johnson, Werner Krandick, and Anatole Ruslanov, Architecture-aware classical Taylor shift by 1. In M. Kauers, editor, International Symposium on Symbolic and Algebraic Computation, pp. 200-207, ACM Press, 2005.

[Richardson09] David Richardson, Efficient Programming Techniques for the SACLIB Computer Algebra Library, Ph.D. Thesis, Drexel University, 2009.

Jeremy Johnson and Lingchuan Meng, Drexel University, USA
[Friday, June 26th, 10:30, B-2624]
SPIRAL-Generated Modular FFTs

In this talk we present a preliminary investigation on the use of the SPIRAL system (www.spiral.net) to generate code for modular Fast Fourier Transforms (FFTs). SPIRAL is a library generation system that automatically generates platform-tuned implementations of digital signal processing algorithms with an emphasis on fast transforms. Currently, SPRIAL can generate highly optimized fixed point and floating-point FFTs for a variety of platforms including vectorization, multi-threaded and distributed memory parallelization. The code produced is competitive with the best available code for these platforms and SPIRAL is used by Intel for its IPP (Intel Performance Primitives) and MKL (Math kernel Library) libraries.

The SPIRAL system uses a mathematical framework for representing and deriving algorithms.Algorithms are derived using rewrite rules and additional rules are used to symbolically manipulate algorithms into forms that take advantage of the underlying hardware. A search engine with a feedback loop is used to tune implementations to particular platforms. New transforms are added by introducing new symbols and their definition and new algorithms can be generated by adding new rules.

We extended SPIRAL to generate algorithms for FFT computation over finite fields. This addition required adding a new data type, several new rules and a new transform (ModDFT) definition. In addition, the unparser (where code is generated) was extended so that it can generate code for modular arithmetic. With these enhancements, the SPRIAL machinery can be applied to modular transforms that are of interest to the computer algebra community. This provides a framework for systematically optimizing these transforms, utilizing vector and parallel computation, and for automatically tuning them to different platforms. In this talk we present preliminary results from this exploration.

Benoit Lacelle and Eric Schost, University of Western Ontario, Canada
[Thursday, June 25th, 16:30, B-2624]
Towards an efficient implementation for the resolution of structured linear system

Lots of linear algebra problems can be reduced to the resolution of a linear system: A.X = B. When they are expressed in such a form, it appears that A often admits a pattern : it is said to be a structured matrix. To accelerate its resolution, one can take advantage of that structure. This talk will present our efficient implementation of the Morf-Bitmead-Anderson algorithm for the inversion of scalar structured matrices : its time complexity is quasi-linear in the size of A. That implementation has been associated with a Newton Iteration taking advantage of the structure of A : it is able to inverse polynomial structured matrices with again a time complexity quasi-linear in the size of A.

WEI PAN, XIN LI AND MARC MORENO MAZA, University of Western Ontario, Canada
[Friday, June 26th, 11:30, B-2624]
Implementing Modular Methods in Maple with the Modpn library

Modpn is a C library offering highly optimized routines for multivariate polynomials arithmetic in prime characteristic. By bridging these low-level and architecture-aware routines to high-level mathematical code, large speed-up factors can be obtained. This is case, for instance, with the module FastArithmeticTools of the RegularChains library in Maple 13, which provides commands for solving systems of non-linear equations.

In this talk, we illustrate how Modpn can be used to efficiently implement modular methods in Maple and take advantage of FFT-based and SLP-based polynomial arithmetic. One goal is to show how to handle the technical difficulties arising in this framework, such as data-conversion overheads, choice of characteristics. We also discuss the determination of thresholds between mixed code and non-mixed code. We conclude by an overview of the current development for multi-core support.

ERIC ROBINSON, GENE COOPERMAN, DANIEL KUNKLE AND JÜRGEN MÜLLER, Northeastern University, USA
[Thursday, June 25th, 17:00, B-2624]
Parallel Disk-Based Computation and Computational Group Theory

The authors have worked together over five years to develop a general methodology for parallel disk based computation. This includes: construction of a permutation representation for Thompson's group, acting on 143,127,000 points; construction of a permutation representation for the Baby Monster group (second largest of the sporadic simple groups), acting on 13,571,955,000 points; and a condensation computation for Fi_23 acting on 11,739,046,176 points that resolves an open problem in the Modular Atlas Project. At heart, these problems are search problems. The work typically required a multi-threaded, distributed program using the 30 nodes and corresponding local disks in parallel. Aggregate disk space used ranged as high as 8 terabytes. The need for the highest efficiency led to the discovery of one new search method, and the re-discovery of several other search methods. This is summarized in a taxonomy of parallel disk-based search algorithms. In addition, a new, open source package is presented that automates the difficult task of developing such parallel disk-based software. Lessons learned about the difficulties of such large computations are also presented.

DANIEL ROCHE, Cheriton School of Computer Science, University of Waterloo, Canada
[Friday, June 26th, 8:30, B-2624]
Memory Efficiency in Polynomial Multiplication

The multiplication of univariate polynomials and multi-precision integers is one of the most basic, well-studied, and widely-used operations in mathematical computing. While asymptotic time complexity correlates with performance, it is well-known that memory access and cache misses also contribute significantly to the running time of computer programs. Unfortunately, all algorithms which achieve sub-quadratic time complexity for multiplication currently require at least O(n) auxiliary storage space for their implementation.

New algorithms are presented which achieve the same time complexity as Karatsuba's and FFT-based algorithms without requiring the use of an auxiliary array for temporary storage. We will discuss some optimizations and other issues encountered in writing a low-level C language implementation of these algorithms with the goal of high performance. The memory usage and cache performance will be compared to other existing software, and the effects on the actual running time will be examined. Finally, we will mention some future application areas and further extensions that could be beneficial.

JAN VERSCHELDE, University of Illinois at Chicago, USA
[Thursday, June 25th, 17:30, B-2624]
Multitasking Polynomial Homotopy Continuation in PHCpack

Homotopy continuation methods to solve polynomial systems scale very well on parallel machines. In this paper we examine its parallel implementation on multiprocessor multicore workstations, using threads. Preliminary timings indicate good speedups for basic pleasingly parallel path tracking jobs. The use of multitasking will lead to more efficient parallel implementations in a multi-tiered approach.

YUZHEN XIE AND MARC MORENO MAZA, University of Western Ontario, Canada
[Friday, June 26th, 9:30, B-2624]
Balanced Dense Polynomial Multiplication on Multi-cores

We present high-performance techniques for FFT-based polynomial multiplication on multi-cores, with a focus on unbalanced input data. We show that balanced data can maximize parallel speed-up and minimize cache complexity for bivariate multiplication. Then, we show how multivariate (and univariate) multiplication can be efficiently reduced to balanced bivariate multiplication.

This approach brings in practice dramatic improvements for unbalanced input data, using the Cilk++} concurrency platform, even if the implementation relies on serial one-dimensional FFTs. This latter property allows us to take advantage efficient non-standard and memory efficient FFT techniques, such as Truncated Fourier Transform, which are hard to parallelize.

12. Nonstandard Applications of Computer Algebra

org: Eugenio Roanes-Lozano, Universidad Complutense de Madrid, Spain
Michael Wester, University of New Mexico, USA
Stanly Steinberg, University of New Mexico, USA

Room: B-3432 and B-3420

Schedule

Thursday June 25th Room B-3432

16:00 – 16:30	E. Roanes-Lozano, A. Hernando and J. Antonio Alonso A Logic Approach to Railway Interlocking Systems using Maple
16:30 – 17:00	Michael Wester, Yuzita Yaacob, Stanly Steinberg Computing Integrals over Polynomially Defined Planar Regions and Curves
17:00 – 17:30	G. Aguilera, J. L. Galán, Y. Padilla, P. Rodríguez Implementation of a Probabilistic Logic with Both Precise and Imprecise Connectives in a CAS
17:30 – 18:00	T. Recio, J. R. Sendra, L. F. Tabera, C. Villarino Parametric Characterization of Hypercircles

Friday June 26th Room B-3420

10:30 – 11:00	Rebecca E. Garcia Constructing and Enumerating Magic Circles and Franklin Magic Circles
11:00 – 11:30	L. J. Hernández, E. García Ruiz, M. Teresa Rivas, V. Marco, E. Sáenz-de-Cabezón, I. Pérez-Moreno and F. J. Sáenz-de-Cabezón A Computer Implementation of the Unity Procedure and its Applications to Arthropod Population Dynamics. A Case Study in the European Grape Berry Moth
11:30 – 12:00	William C. Bauldry Data Envelopment Analysis with Maple 13
12:00 – 12:30	Antonio Hernando A New Algebraic Model for Implementing Expert Systems Represented Under the Frames-Paradigm

Abstracts

Gabriel Aguilera, José Luis Galán, Yolanda Padilla, Pedro Rodríguez, University of Málaga, Spain
[Thursday, June 25th, 17:00, B-3432]
Implementation of a Probabilistic Logic with Both Precise and Imprecise Connectives in a CAS

In this work a new Probabilistic Logic, PL≈, is described in a semantic and in a syntactic way. One of the main characteristic of this logic is the use of both, precise and imprecise connectives.

In order to allow imprecise probabilities, each formula from PL≈ is provided with an interval that reflects the minimum and maximum probabilities possible for this formula. These two real numbers in [0, 1] interval can be stored in a standard float point notation but for the calculus of precise connectives an exact representation of the real numbers is needed. Thus, the implementation of the semantic of precise connectives of PL≈ requires a CAS tool, since an exact calculus of the probabilities is needed which it is not supported by traditional float point languages.

Syntactic rules of the logic PL≈ will be implemented in a CAS to allow the computer to simplify formulae and to obtain results.

As an example of application of this logic some examples involving card games will be implemented. Precise connectives will be used, for instance, for the calculus of traditional probabilities like the probability of obtaining a better result than the opponent. Imprecise connectives and imprecise probabilities will be used for the calculus of the probability of the event “opponent is bluffing”.

WILLIAM C. BAULDRY, Appalachian State University, USA
[Friday, June 26th, 11:30, B-3420]
Data Envelopment Analysis with Maple 13

We will present the ``Data Envelopment Analysis'' (DEA) technique using Maple 13. DEA is an operations research tool that uses the simplex algorithm to evaluate the relative efficiency of `decision- making units' (DMU's) in an organization. We will illustrate DEA by applying the method to analyze the 16 departments of the College of Arts & Sciences at Appalachian State University. Computer algebra is best at easily handling the large scale computations intrinsic to DEA studies. The College of Arts & Sciences DEA analysis, a simplified example, requires 16 simplex optimizations each over 16 variables with 3 constraints. A significant DEA study would likely involve 30 or more simplex optimizations with a much larger number of variables and of constraints for each run. The original Input-Output ratio form of DEA is nonlinear and is quite difficult to compute; Maple makes it feasible to attempt.

REBECCA E. GARCIA, Sam Houston State University, USA
[Friday, June 26th, 10:30, B-3420]
Constructing and Enumerating Magic Circles and Franklin Magic Circles

There are several variations on magic circles. One famous magic circle was constructed by Benjamin Franklin, circa 1752, which is an arrangement of nonnegative integers in a circular grid consisting of eight concentric annuli and eight radial segments. Franklin's circle had many properties, including the standard magic property: that the annular sum and the radial sum equal the same magic number M. Franklin's magic circle is an example of what we call an r-magic 8-circle.

Another similar type of magic circle is what we call a d-magic n-circle, which is an arrangement of nonnegative integers in a circular grid consisting of n concentric annuli and n diametrical segments, where the annular sum and the diametrical sum equal the same magic number M.

In this presentation, we discuss some techniques in computational algebraic combinatorics and enumerative geometry to construct and to count these variations on magic circles. We provide a very nice description of their minimal Hilbert basis, which is useful in determining the symmetry operations on magic circles and, consequently, in enumerating natural magic circles. Finally, we present the enumerating functions for the Franklin magic 8-circles, the r-magic circles, and the d-magic circles.

ANTONIO HERNANDO, Universidad Politécnica de Madrid, Spain

[Friday, June 26th, 12:00, B-3420]

A New Algebraic Model for Implementing Expert Systems Represented Under the Frames–Paradigm

This paper is concerned with expounding a new representation paradigm for modelling expert systems based on computing Groebner Bases. Previous research on Groebner Bases expert systems has so far been connected to modelling expert systems based on (both bi- and multi-valued) propositional logics. Our approach instead is based on the well-known Artificial Intelligence frames paradigm for representing knowledge. More precisely, our research is based on translating an already existent expert system described in terms of the frames paradigm to a new algebraic model which represents knowledge by means of polynomials. In this way, issues about consistence and inference within this expert system will be, through this new model, transformed into algebraic problems involving calculating Groebner Bases.

By using this new model of ours, some interesting advantages ensue: on the one hand, knowledge representation may be performed in a more straightforward and intuitive way; on the other, calculating the Groebner Bases associated to our algebraic model is usually faster adopting this new frames-based paradigm than it was in previous propositional logic-based expert systems.

The exposition is illustrated with some interesting examples in which the main advantages of our model are established by comparing its performance to that of the mentioned propositional logic-based expert systems.

Acknowledgments

This work was partially supported by the research project UCM2008-910563 (UCM - BSCH Gr. 58/08, research group ACEIA).

L. Javier Hernández, Esteban García Ruiz, M. Teresa Rivas, Vicente Marco, Eduardo Sáenz-de-Cabezón, Ignacio Pérez-Moreno and F. Javier Sáenz-de-Cabezón, Universidad de La Rioja and I.N.I.A., Spain

[Friday, June 26th, 11:00, B-3420]

A Computer Implementation of the Unity Procedure and its Applications to Arthropod Population Dynamics. A Case Study in the European Grape Berry Moth

A new procedure based on partition of the unity has been developed to construct spatial approximation-prediction functions for discrete density functions. This Partition of the Unity Procedure (PUP) provides a family of approximation-prediction functions which depends on several parameters. In order to find the best function we have considered error estimators induced by the Vietoris simplicial set associated to an influence radius.

A computational implementation of these mathematical models has been elaborated to be applied to any kind of finite discrete data to obtain the parameters which minimize the approximation and prediction error.

These mathematical and computational methods (completed with biological, cultural, physical and chemical studies) are applied to study the population dynamics of the European grape berry moth Lobesia botrana, key pest of European and Mediterranean vineyards. The main goal of this approach is to develop tools for a better understanding and management of this and other important pests. In our particular case study this is translated in the development of an Integrated Pest Management (IPM) program for the L. botrana in La Rioja in order to minimize economic, health and environmental risks.

The goal of our work is then, twofold: On one side we develop new procedures to construct approximation-prediction functions, and on the other side we give an example on the implementation of these methods for the study and management of L. botrana, developing a software accessible to the agents involved (from farmers to researchers).

The new technique can be applied to study different components of the population dynamics of arthropods in general. In the L. botrana case we studied (i) the analysis of the spatial distribution using density functions obtained by adults catches, (ii) the analysis of the development rate of the pest in the different parts of La Rioja, and (iii) the analysis of the spatiotemporal distribution and the calculus of the annual number of generations. This contribution focuses on the first parts being the analysis of the spatiotemporal distribution work under development from which interesting results have already been obtained.

T. Recio, J. R. Sendra, L. F. Tabera, C. Villarino, Universidad de Cantabria, Spain, Universidad de Alcalá, Spain, and University of Berkeley, USA

[Thursday, June 25th, 17:30, B-3432]

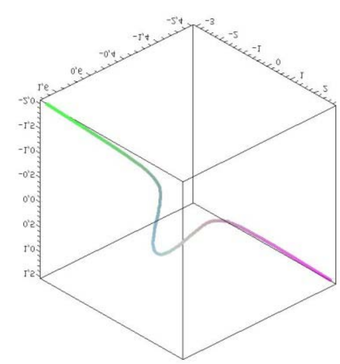
Parametric Characterization of Hypercircles

We can think of the real plane as the field of complex numbers $\mathbb{C}$, an algebraic extension of the reals $\mathbb{R}$ of degree 2. Analogously, we can consider a characteristic zero base field $\mathbb{K}$ and an algebraic extension of degree n,

$\mathbb{K}(\alpha)=\mathbb{K}[\alpha]$, i.e. expressions given by $\mathbb{K}$-polynomials in α up to degree $ n-1$ (which turns to include as well all quotients of such expressions, exactly in the same way as a quotient of two complex numbers --a $\mathbb{R}$-polynomial of degree 1 in the letter i-- is again a complex number). Let us identify $\mathbb{K}(\alpha)$ as the vector space $\mathbb{K}^n$, via the choice of a suitable base, such as the one given by the powers of α.

Then, recall that a real circle can be defined as the image (in the real plane, suitably identified with the complex numbers) of the real axis under a Moebius transformation (of the kind $\frac{at+b}{ct+d}$, with $a,b,c,d \in \mathbb{C}$) in the complex field. Likewise, and roughly speaking, a hypercircle (i.e. a {\it non-standard circle}) can be defined as the curve in $\mathbb{K}^n$ that is the image of ``the ${\mathbb{K}}$-axis" under the transformation $\frac{at+b}{ct+d}:\mathbb{K}(\alpha)\rightarrow\mathbb{K}(\alpha)$. They have been introduced in \cite{ARS-2} and studied in detail in \cite{Generalizing-circles}.

For example, if we take $\mathbb{K}=\mathbb{Q}$ and $ \alpha$ such that $\alpha^3+2=0$, and we consider the map $\Phi= \displaystyle{\frac{t + \alpha}{t - \alpha}}$, we obtain the hypercircle in $\mathbb{Q}^3$ parametrized (i.e. obtained as the image of a mapping from $\mathbb{Q} \longrightarrow \mathbb{Q}^3$ defined) by $ \displaystyle{[\frac{t ^3-2}{2+t^3}, \frac{2t^2}{2+t^3}, \frac{2t}{2+t^3}]}$, with plot as follows



The study of these hypercircles is fascinating and opens the door to stating many different questions. For instance, circles, through classical Moebius transformations, are related to conformal (=angle perserving) geometry and to complex holomorphic (i.e. functions of one complex variable are complex-differentiable at every point) functions. Which analogous notions of angle-perserving and holomorphic functions could be defined through the general framework that allows the definition of hypercircles? We think that the possibility of connecting that part of Mathematics to Computer Algebra is, again, highly {\it non-standard}.

In this direction, we will focus in our communication on a humble and basic problem. Given an algebraic extension of degree n, $\mathbb{K}(\alpha)$, and a parametric curve in $\mathbb{K}^n$, when is it a hypercircle? And, if so, we want to algorithmically identify the transformation $\frac{at+b}{ct+d}:\mathbb{K}(\alpha)\rightarrow\mathbb{K}(\alpha)$ yielding the hypercircle. Notice that, for the classical case of standard circles, the problem is to determine if a plane curve, given by a parametrization (perhaps with complex coefficients), is a circle and, in the affirmative case, to find its geometric elements, since they determine the Moebius transformation.

We will present a complete and algorithmic solution to both questions for hypercircles and will briefly comment on the following application of this (on the other hand quite natural) problem. Assume a planar or spatial rational curve $\mathcal{C}$ is given by a parametrization over $\mathbb{K}(\alpha)$. Then, we want to obtain, whenever possible, a simpler parametrization over $\mathbb{K}$ of the same curve $\mathcal{C}$. In \cite{ARS-2} it is shown that this problem is reduced to determining that a certain curve is a hypercircle. Moreover, if we have a $\mathbb{K}$ parametrization of this hypercircle, a $\mathbb{K}$ parametrization of the original curve is then achieved by a simple substitution. This may-be hypercircle is found manipulating algorithmically the parametrization of the originally given curve, by a method analogous to Weil's descent (see \cite{Weil} for a detailed description of this procedure).

So, the communication we propose here contributes to closing the solution to this simplification problem, since it allows to algorithmically decide if a given curve is a hypercircle and to parametrize it over $\mathbb{K}$, which is trivial once the corresponding "Moebius" transformation is known.

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EUGENIO ROANES-LOZANO, ANTONIO HERNANDO AND **JOSE ANTONIO ALONSO**, Universidad Complutense de Madrid, Universidad Politécnica de Madrid and Universidad de Sevilla, Spain
[Thursday, June 25th, 16:00, B-3432]

A Logic Approach to Railway Interlocking Systems using Maple

Railway interlocking systems are apparatuses that prevent conflicting movements of trains through an arrangement of tracks. An interlocking takes into consideration the position of the switches (of the turnouts) and does not allow trains to be given clear signals unless the routes to be used by the trains do not intersect. The authors had previously developed matrix-based and algebraic (Groebner bases) models for the same goal. These models are independent from the topology of the station.

Now a new model, based on Boolean logic, has been developed. Its main contribution can be summarily described as follows: according to this new model, any given proposed situation is safe iff a certain set of formulae (translating the position of trains and the movements allowed –the latter depend on the position of the switches and the color of the semaphores) is consistent. This model is independent from the topology of the station too.

The package has been implemented in the computer algebra system (CAS) Maple, and it makes an extensive use of its ”Logic” package and the facilities provided by this CAS to operate with sets. The code is surprisingly brief. The fact that trains could occupy more than one section is considered.

The main procedure analyzes the safety of a proposed situation and returns, if they exist, the sections where a collision could take place. Another procedure checks whether a given section is accessible by a train located on other given section or not.

Regarding execution times, the model shows similar times to the previous matrix and GB-based models in intermediate situations. Nevertheless, their speed evolve in very different ways. The GB-based one is very fast when there are many trains and many accesses to other sections are allowed to those trains. Meanwhile, the model presented here is very fast when there are few allowed “itineraries” (what would be called paths in Graph Theory).

Acknowledgments

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MICHAEL WESTER, YUZITA YAACOB, STANLY STEINBERG, University of New Mexico, USA, and National Univ. of Malaysia, Malaysia

[Thursday, June 25th, 16:30, B-3432]

Computing Integrals over Polynomially Defined Planar Regions and Curves

We use the cylindrical algebraic decomposition algorithm implemented in Mathematica to produce algorithms to analytically compute integrals over polynomially defined regions in the plane. We also produce an algorithm to compute integrals over the boundary of such regions and then implement the two dimensional version of the Green's theorem. The resulting software can solve many of the two dimensional integration problems in calculus textbook.

13. Symbolic and numeric approaches to dynamical modeling and simulation

org: Jürgen Gerhard, Maplesoft, Canada
Austin Roche, Maplesoft, Canada
Elena Shmoylova, Maplesoft, Canada

Room: B-4404

Schedule

Thursday June 25th

10:30 – 11:00	Jürgen Gerhard Industrial Dynamical Modeling and Simulation
11:00 – 11:30	Marc Moreno Maza Regular Chains and Differential Elimination
11:30 – 12:00	Gregory Reid Numeric-Geometric Techniques for Differential Equations I. Introduction
12:00 – 12:30	Niloofar Mani Numeric-Geometric Techniques for Differential Equations II. Applications
14:00 – 14:30	Allan Wittkopf – talk presented by Austin Roche Symbolic Preprocessing of DAE Systems
14:30 – 15:00	Henrik Tidefelt Unstructured matrix-valued singular perturbations -- tackle or avoid?
15:00 – 15:30	Thomas Wolf Applications of the package CRACK to simplify large systems
16:00 – 16:30	Benjamin J. Spivey, John D. Hedengren* and Thomas F. Edgar Monitoring of Process Fouling Using First Principles Monitoring and Moving Horizon Estimation
16:30 – 17:00	Erik Postma and Elena Shmoylova* Computer Algebra versus Reality
17:00 – 17:30	Thomas Wolf A hybrid discrete-polynomial dynamical system modeling board positions in the game of go
17:30 – 18:00	Jay Bardhan Challenges in Coupling Simulation and Optimization: Biomolecule Design

Abstracts

Jay Bardhan, Argonne National Laboratory and Rush University, USA
[Thursday, June 25th, 17:30, B-4404]
Challenges in Coupling Simulation and Optimization: Biomolecule Design

In this talk I will present an PDE-constrained approach to optimizing the electrostatic interactions between biological molecules, a problem relevant to the analysis and design of new drugs as well as to advancing our understanding of the design strategies employed by biological systems. Our research in this area serves as a case study to highlight some of the conceptual challenges involved in coupling modeling approaches such as simulation and optimization. Here, the simulations are partial-differential equation (PDE)-based mathematical models and the objective function is a relatively simple quadratic.

Re-engineering the interface between simulation and optimization drastically reduces the time required to find an optimal solution, but complicates the task of regularization. The need to address this complication has led, in turn, to our recent discovery of a promising new approach for modeling electrostatic interactions between molecules.

Jürgen Gerhard, Maplesoft, Canada
[Thursday, June 25th, 10:30, B-4404]
Industrial Dynamical Modeling and Simulation

Many engineering companies have been routinely employing dynamical modeling and simulation in their day-to-day design work. This talk will briefly present three projects related to modeling and simulation that Maplesoft is involved in, discuss some of the main challenges that designers and users of modeling and simulation tools are facing, and present some example models from industrial applications.

Niloofar Mani, University of Western Ontario, Canada
[Thursday, June 25th, 12:00, B-4404]
Numeric-Geometric Techniques for Differential Equations II. Applications

There are software packages that support high-level physics-based modeling and simulation. One of the latest is MapleSim which allows you to build component diagrams that represent physical systems in a graphical form. Using both symbolic and numeric approaches, this software automatically generates model differential equations with constraints (so-called differential-algebraic equations or DAE) from a component diagram and runs high-fidelity simulations.

We describe initial steps towards using geometric methods including homotopy continuation to analyze and help solve such systems numerically. This talk is a sequel to the introductory talk by Greg Reid, earlier in this session. The geometric methods have the advantage of numerical stability compared with symbolic differential-elimination methods, which much like Gauss elimination, can be unstable due to pivots on small approximate quantities. The talk will be illustrated by examples and applications.

MARC MORENO MAZA, Massachusetts Institute of Technology, USA
[Thursday, June 25th, 11:00, B-4404]
Regular Chains and Differential Elimination

Regular chains are one of the major tools for solving polynomial systems. For systems of algebraic equations, they provide a convenient way to describe complex solutions and a step toward isolation of real roots or decomposition into irreducible components. Combined with other techniques, they are used for these purposes by several computer algebra systems.

For systems of partial differential equations, they provide a popular way for determining a symbolic description of the solution set. Moreover, thanks to Rosenfeld's Lemma, techniques from the algebraic case apply to the differential one

In this talk, we first review the fundamental differential operations that, in practice, rely directly on this reduction to the algebraic case, namely pseudo-division, regularity test, regular GCDs and ranking conversions in some cases. Then, we discuss how the recent improvements of the algebraic operations (based on modular methods, fast polynomial arithmetic, parallel algorithms) can benefit to their differential counterparts.

ERIK POSTMA AND ELENA SHMOYLOVA*, Maplesoft, Canada
[Thursday, June 25th, 16:30, B-4404]
Computer Algebra versus Reality

Many of the symbolic methods prevalent in dynamical modeling were designed specifically for a certain class of "nice" models, such as, e.g., polynomials with integer coefficients. However, in practice many models contain components that are not easily accessible to purely symbolic manipulations, such as floating point coefficients and exponents, lookup tables, or piecewise defined functions. The challenge is to apply symbolic techniques to such models appropriately and effectively.

In this presentation we would like to list some of the problems that are often encountered when dealing with real world applications. We then present some possible solutions applicable in some cases. Most importantly, we would like to initiate a discussion with the audience on how to approach these issues.

GREGORY REID, University of Western Ontario, Canada
[Thursday, June 25th, 11:30, B-4404]
Numeric-Geometric Techniques for Differential Equations I. Introduction

This is the first of two talks about stable numeric-geometric methods for general systems of differential equations with constraints (so-called differential-algebraic equations or DAE). Such systems are attracting much attention since they are automatically generated by computer modeling environments such as MapleSim. Determination of such constraints is essential for the determination of consistent initial conditions and the numerical solution of such systems. This talk will concentrate on introduction of concepts from the (Jet) geometry of differential equations, illustrated by visualizations and simple examples. A subsequent talk by Niloofar Mani, will discuss initial investigations that we have made using MapleSim, and such approaches.

This talk will be an introduction to stable numerical methods for such general systems. The corresponding problem for the non-differential case, that of approximate polynomial systems, has only recently been given a solution, through the works of Sommese, Wampler and others. The new area called numerical algebraic geometry, will also be described. Key data structures are certain witness points on jet manifolds of solutions, computed by stable homotopy continuation methods.

Benjamin J. Spivey, John D. Hedengren* and Thomas F. Edgar, University of Texas and ExxonMobil Chemical Company, USA
[Thursday, June 25th, 16:00, B-4404]
Monitoring of Process Fouling Using First Principles Monitoring and Moving Horizon Estimation

Chemical reactor fouling is a widely recognized challenge for maintaining production rates and preventing unplanned downtime in industrial plants. While the extent of fouling is difficult to measure directly, a long-term effect of fouling on the process may be evident by incorporating process knowledge with indirect measurements. A primary shortcoming of this heuristic fouling indicator is correlation between indirect measurements and continually changing process and ambient conditions.

This talk presents a moving horizon estimation (MHE) approach to estimating fouling parameters. The estimation approach utilizes first principles and empirical elements to define the process model. Preliminary results indicate the quantitative fouling indicator tracks with heuristic process observations and provides a consistent measure of fouling. This indicator enables production decision makers to quantify the effects of process improvements in order to reduce fouling rates over extended periods of time under varying conditions.

Henrik Tidefelt, Linköping University, Sweden
[Thursday, June 25th, 14:30, B-4404]
Unstructured matrix-valued singular perturbations -- tackle or avoid?

Quasi-linear differential-algebraic equations is a convenient model structure for dynamical systems. Such models generally contain both exactly known and uncertain coefficients. The structure that the exactly known coefficients adds to the equations can be utilized when analyzing or solving the equations, but requires a more detailed model structure than general DAE. When developing theory and/or software for DAE, one needs to decide what additional structure to impose on the equations, and it is tempting to take on as general forms as possible, both to gain wide applicability and to avoid the extra bookkeeping that any additional structure would require.

In this talk, the consequences of neglecting the structure that exactly known coefficients bring to the equations will be discussed. The resulting problem will also be motivated by symbolic-numeric approaches to integration of exactly known systems, and a general approach to tackle it will be presented. In view of available results, it is also motivated to question the use of unstructured DAE as a model structure.

ALLAN WITTKOPF, Maplesoft, Canada -- Presented by **AUSTIN ROCHE**, Maplesoft, Canada
[Thursday, June 25th, 14:00, B-4404]
Symbolic Preprocessing of DAE Systems

In this talk we will discuss the motivation for use of a symbolic engine, such as Maple, as a preprocessor for numerical ODE system solution. Some issues relating to scalability for larger models will be discussed, including derivative ranking (solving order), redundancy equation detection, and use of implicit form subsystems (for dense parts of the input system).

THOMAS WOLF, Brock University, Canada
[Thursday, June 25th, 15:00, B-4404]
Applications of the package CRACK to simplify large systems

In the three applications

- the computation of quadratic Poisson Structures,
- the construction of Gardner's deformation for the Bosonic Limit of the N=2 supersymmetric Burgers equation, and
- the computation of a Killing tensor for the Kimura metric

an overdetermined algebraic or differential system has been formulated, simplified and fully or partially solved. After a short introduction of the problems and their specific challenges, modules of the computer algebra package CRACK are introduced that were used in the computations. In some cases CRACK was used to pre-simplify the problem before it was solvable by other packages, like Singular.

THOMAS WOLF, Brock University, Canada
[Thursday, June 25th, 17:00, B-4404]
A hybrid discrete-polynomial dynamical system modeling board positions in the game of go

It is well known that the game of go poses a special challenge to computer technology, from modeling to programming, from Artificial Intelligence to hardware. Despite three decades of world wide effort and recent progress with Monte Carlo type go programs it is still possible for young children of Dan level playing strength to beat the strongest programs running on clusters with 100s of nodes.

The talk will describe the dynamic creation of a model that aims at describing board positions in go. The model exploits the partially local nature of go (more precisely of the capture rule in go) and leads in its simple version to a polynomial dynamical systems for over 300 unknowns which to formulate needs computer algebra support. Better models require the solution of hybrid discrete-polynomial systems.

The talk will further report on the numerical solution of these dynamical systems. A demo shows the real time operation of such a model and compares its move predictions to moves made in professional games.

14. Algorithms for Parametric Systems and their Applications

org: Guillaume Moroz, Maplesoft, Canada
Hirokazu Anai, Fujitsu Laboratories Ltd/Kyushu University, Japan

Room: B-3432

Schedule

Thursday June 25th

10:30 – 11:00	AKIRA SUZUKI Groebner bases computation within linear algebra and its application to comprehensive Groebner systems
11:00 – 11:30	ERIC SCHOST, XAVIER DAHAN AND ABDULILAH KADRI Bit-size bounds for regular chains in positive dimension
11:30 – 12:00	GIOVANNA CORAL AND LAUREANO GONZALEZ-VEGA Algorithms for hyperbolic and trigonometric curves: implicitization and parameterization
12:00 – 12:30	GUILLAUME MOROZ Groebner bases and parametrization
14:00 – 14:30	CHANGBO CHEN AND MARC MORENO MAZA Solving Parametric Polynomial Systems with the RegularChains Library in Maple
14:30 – 15:00	JAMES DAVENPORT Simpler Interfaces to Complicated Concepts
15:00 – 15:30	JAN VERSCHELDE Sweeping for singular solutions of polynomial systems with parameters

Abstracts

CHANGBO CHEN AND MARC MORENO MAZA, University of Western Ontario, Canada and MIT, USA
[Thursday, June 25th, 14:00, B-3432]
Solving Parametric Polynomial Systems with the RegularChains Library in Maple
(Changbo Chen, Francois Lemaire, Marc Moreno Maza, Bican Xia, Rong Xiao, Yuzhen Xie)

Solving systems of parametric polynomial equations symbolically is in demand for an increasing number of applications such as program verification, optimization and the study of dynamical systems. Groebner bases and triangular decompositions are classical techniques for processing parametric systems. Recent research has focused on enhancing theories and algorithms to meet the practical requirement of these systems.

The ParametricSystemTools module of the RegularChains library in Maple implements comprehensive triangular decompositions (CTD) and real root classification (RRC) which are tools dedicated to study polynomial systems with parameters. It is supported by the modules ConstructibleSetTools and SemiAlgebraicSetTools. The first one provides useful commands for solving over the complex numbers and the second over the reals.

This talk is an overview of the functionalities of these three modules. We start with a review of the fundamental concepts of CTD, RRC and Border Polynomial together with their relation with other popular notions for parametric polynomial systems. We consider then applications from program verification and biochemistry. In this latter case, we will combine our library with software tools for modeling.

GIOVANNA CORAL AND LAUREANO GONZALEZ-VEGA, Universidad de Cantabria, Spain
[Thursday, June 25th, 11:30, B-3432]
Algorithms for hyperbolic and trigonometric curves: implicitization and parameterization

A hyperbolic poynomial is defined in the following way: $\sum_{k=0}^{m} a_k cosh(k)+\sum_{k=0}^{m} b_k sinh(k)$, where $a_k \in R$ and $b_k \in R$. A hyperbolic curve is a real plane curve where each coordinate is given parametrically by a hyperbolic poynomial:

$$x=\sum_{k=0}^{m} a_k cosh(k)+\sum_{k=0}^{m} b_k sinh(k)$$

$$y=\sum_{k=0}^{m} c_k cosh(k)+\sum_{k=0}^{m} d_k sinh(k)$$

By adapting to hyperbolic curves the algorithms presented in [Hong and Schicho 98] for the trigonometric case, we give algorithms for simplifying a given parametric representation and for computing an implicit representation from a given parametric representation.

We show moreover that some of the algebraic curves arising from the implicitization of a hyperbolic curve have a very special structure containing both one hyperbolic part and one trigonometric part. For example:

$$2752x^2-32x^2y+2x^4+310632-172y-130y^2-y^3=0$$

contains two curves, one trigonometric and the other hyperbolic.

JAMES DAVENPORT, University of Waterloo, Canada
[Thursday, June 25th, 14:30, B-3432]
Simpler Interfaces to Complicated Concepts
(Changbo Chen, James H Davenport, John May, Marc Moreno Maza, Bican Xia, Rong Xiao and Yuzhen Xie)

As computer algebra develops, it handles more sophisticated objects, many of which have no precise parallel in conventional mathematics, since mathematicians have handled the concepts on an ad hoc basis. Furthermore, by definition, computer algebra must handle these objects algorithmically, and present them to the user. This is particularly a challenge when the user may not be intimately familiar with the object, and all the special cases that may occur.

We present various issues connected with this in the context of equation solving, and show how the 'piecewise' construct of Maple can be employed to build representations of solution objects that:

1. Are intuitive in simple cases;
2. familiar base constructs;
3. Allow `delayed evaluation' of difficult special cases, which the user may not actually be interested in.

GUILLAUME MOROZ, Maplesoft, Canada
[Thursday, June 25th, 12:00, B-3432]
Groebner bases and parametrization

Let S be a polynomial system of equations in $X_1,\ldots,X_n$. Furthermore, assume that its coefficients depend on the symbolic parameters $T_1,\ldots,T_s$. A natural problem in some applications is to compute a parametrization of the solutions of S.

Under some assumptions, the solutions of S can be written under the shape:

$$X_1=Q_1(T_1,\ldots,T_s,Z),\ \ldots,\ X_n=Q_n(T_1,\ldots,T_s,Z) \text{ and } P(T_1,\ldots,T_s,Z)=0$$

where $Q_1,\ldots,Q_n$ are rational functions, P is a polynomial and Z is a new symbolic variable.

We will give an overview of different methods computing such a parametrization. Then we will present a parametrization based on Groebner basis computation for a specific product order, and show the advantages of this representation on some examples.

ERIC SCHOST, XAVIER DAHAN AND ABDULILAH KADRI, University of Western Ontario, Canada
[Thursday, June 25th, 11:00, B-3432]
Bit-size bounds for regular chains in positive dimension

Extending previous results by Dahan and Schost, we show that for an algebraic set V of positive dimension, the bit-length of the coefficients of some suitably normalized regular chains representing V is polynomially bounded in terms of the degree and height of V.

AKIRA SUZUKI, Kobe University, Japan
[Thursday, June 25th, 10:30, B-3432]
Groebner bases computation within linear algebra and its application to comprehensive Groebner systems

We introduce an algorithm to compute Groebner bases within linear algebra. For a given finite set of polynomials, it compute both an appropriate term ordering and the corresponding reduced Groenber basis of the ideal generated by the given polynomials.

Though our original algorithm changes term orderings dynamically for computational efficiency, it is also possible to change them suitable for the Suzuki-Sato algorithm, which compute comprehensive Groebner basis. In this talk, we argue on the algorithms and its implementations.

JAN VERSCHELDE, University of Illinois, USA
[Thursday, June 25th, 15:00, B-3432]
Sweeping for singular solutions of polynomial systems with parameters
(Jan Verschelde, Kathy Piret)

Many problems give rise to polynomial systems. These systems often have several parameters and we are interested to study how the solutions vary when we change the values for the parameters. Using predictor-corrector methods we track the solution paths. A point along a solution path is critical when the Jacobian matrix The simplest case of quadratic turning points is well understood, but these methods no longer work for general types of singularities. We have experimented with criteria to monitor the Jacobian in order not to miss any singular solutions along a path. In case of higher order singularities more accurate predictors are needed, otherwise we do not get in the range for which reconditioning methods such as deflation can be applied. Our methods are implemented in the software package PHCpack and applied to a wide range of polynomial systems arising in various fields of science and engineering. This is joint work with Kathy Piret.

CONTENTS