Application of CAS to the classical definition of limit of function

M. A. Bokhari[1] and B. Yushau[2]
[1]Department of Mathematics & Statistics
[2]Prep-Year Math Program
King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia

Abstract

The limit of a real valued function is a fundamental concept in calculus. However, most of math major and engineering students conceive the classical definition of limit as the most problematic part of calculus. They consider the $\varepsilon-\delta$ definition of limit difficult to understand. In particular, finding the largest value of δ for a given $\varepsilon>0$ in case of simple non-linear functions like $f(x)=x^2$ turns out challenging for them due to involvement of inequalities. Many students find this definition of no use and therefore, skip it. On the other hand, the role of $\varepsilon-\delta$ definition is inevitable in case of justifying some assertions like "$\lim_{x\to 0}\sin(1/x)$ *does not exist*". The basic calculus books suggest the use of calculators or CAS in order to validate this type of statements because the students are usually unable to follow contra-positive arguments to justify such statements at the freshman level.

In 2006, we reformulated $\varepsilon-\delta$ definition in terms of local (L-ε) approximation for a single variable function f. It appeared in IJMEST (V.37,No. 5, 2006, p. 515-526). Our approach for finding the value of δ is based on computing the real zeros of two fucntions

$$\left.\begin{aligned} g_1(x)&=f(x)-L+\varepsilon \\ g_2(x)&=f(x)-L-\varepsilon \end{aligned}\right\}. \qquad (1)$$

This, in case of non-linear functions like $f(x)=x^n$, $f(x)=ax^n/(bx^n+c)$ or $f(x)=\cos(ax+b)$ etc, is straightforward to handle manually or with a simple scientific calculator. Nevertheless, an appropriate mathematical software is required for estimating the real zeros for several types of functions like $f(x)=(x^3-2x^2+3x+5)/(x^2+9)$ or $f(x)=\cos(x)/\ln(x-\pi+e)$ etc.

The objective of our talk is two fold. We shall

(i) demontrate the use of various software to the functions $g_i(x), i=1,2$ (cf (1)) by considering a variety of examples and compare their effectiveness in estimating the largest value of δ.

(ii) explain an extension of the notion of local (L-ε) approximation to functions of two variables and discuss application of CAS for estimating a vlue of δ for quadric functions.