Technology in mathematics education:

from meaning to purpose

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If it had been possible to develop graphics first…

… this is how we might have started using computers in mathematics courses.

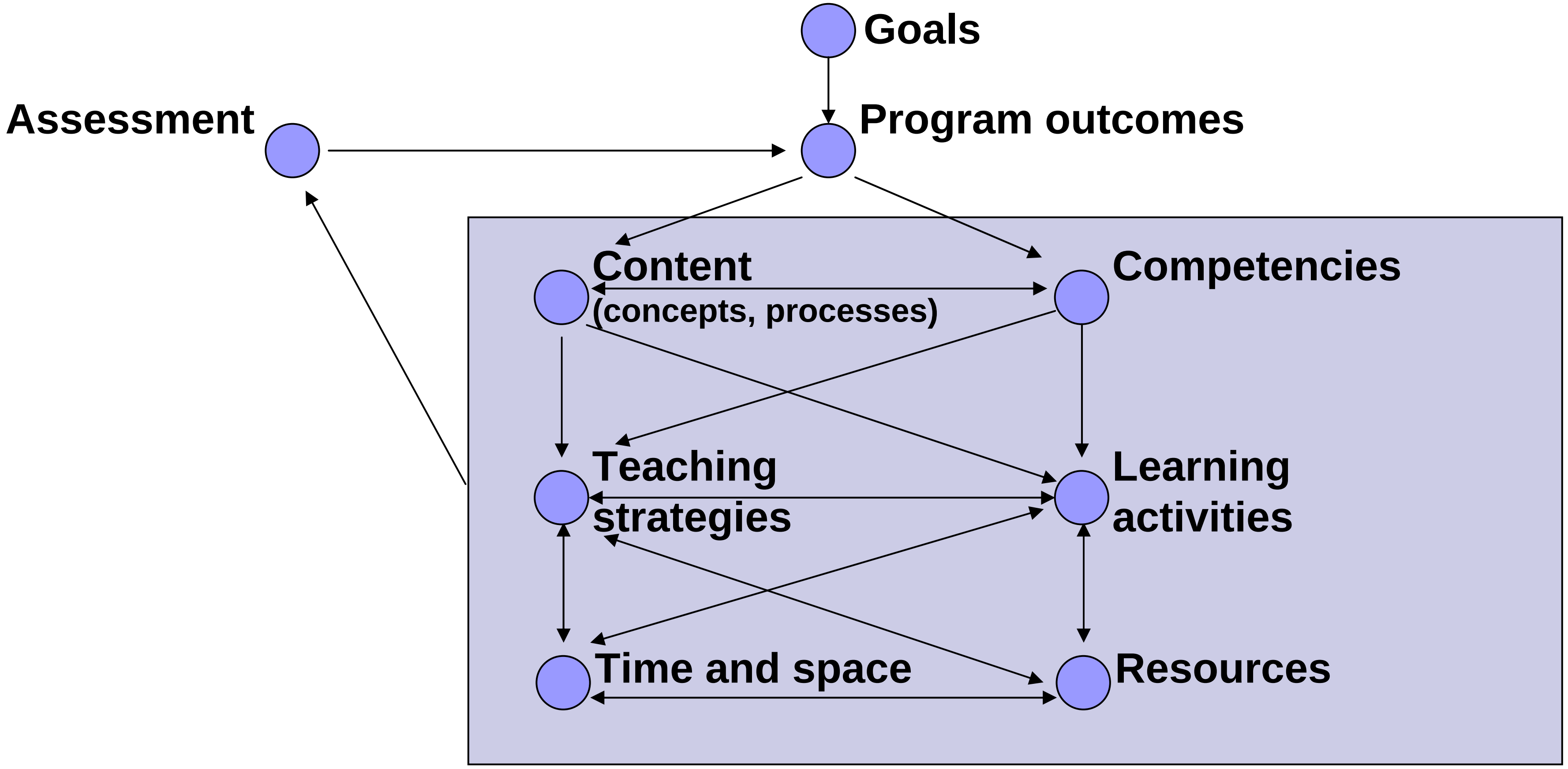
A fondamental question

What are we trying to achieve with the integration of technology in the teaching and learning of mathematics?

A related question:

What are we trying to achieve with the teaching of mathematics?

A curriculum perspective

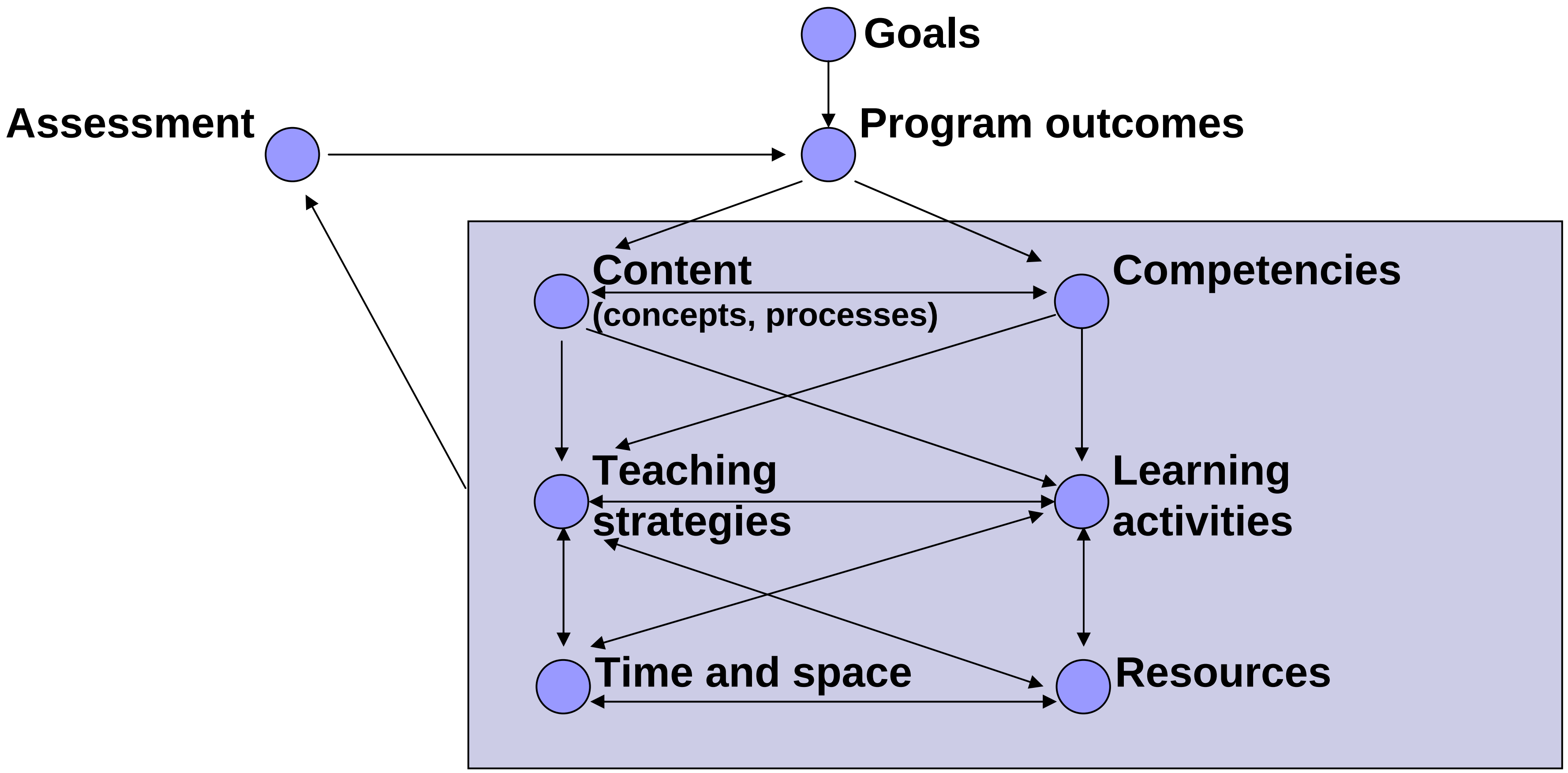


Where should we start?

The « safe approach » (Chevallard, 1992):

- initially limit the role of technology to improving the teaching and learning of traditional mathematical content;
- then look into the implications on mathematical knowledge of using the tool.

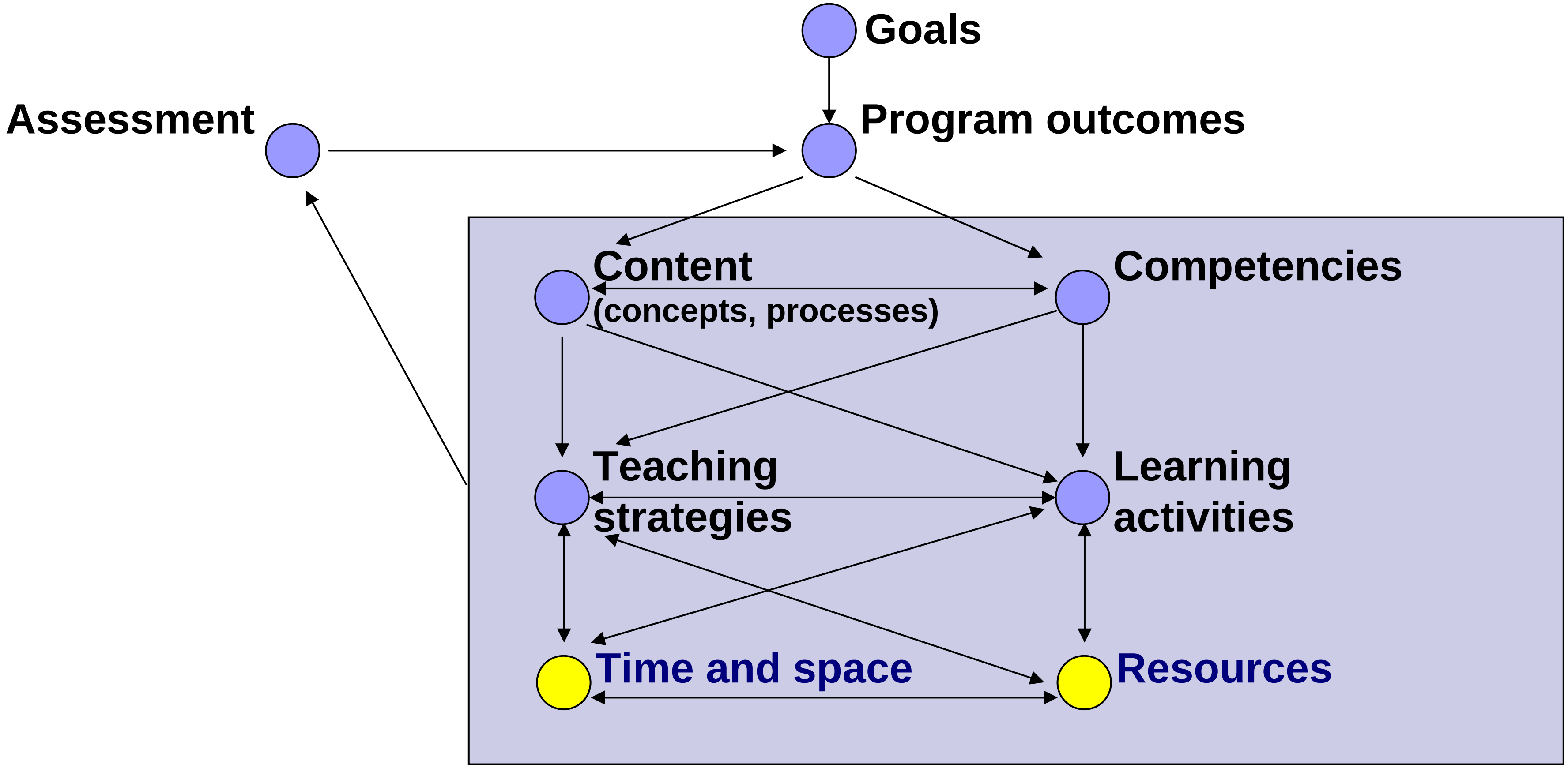
Why use CAS ?



Goals
Assessment
Program outcomes
Content
(concepts, processes)
Competencies
Teaching
strategies
Learning
activities
Time and space
Resources

Why use CAS ?

Because it’s there.

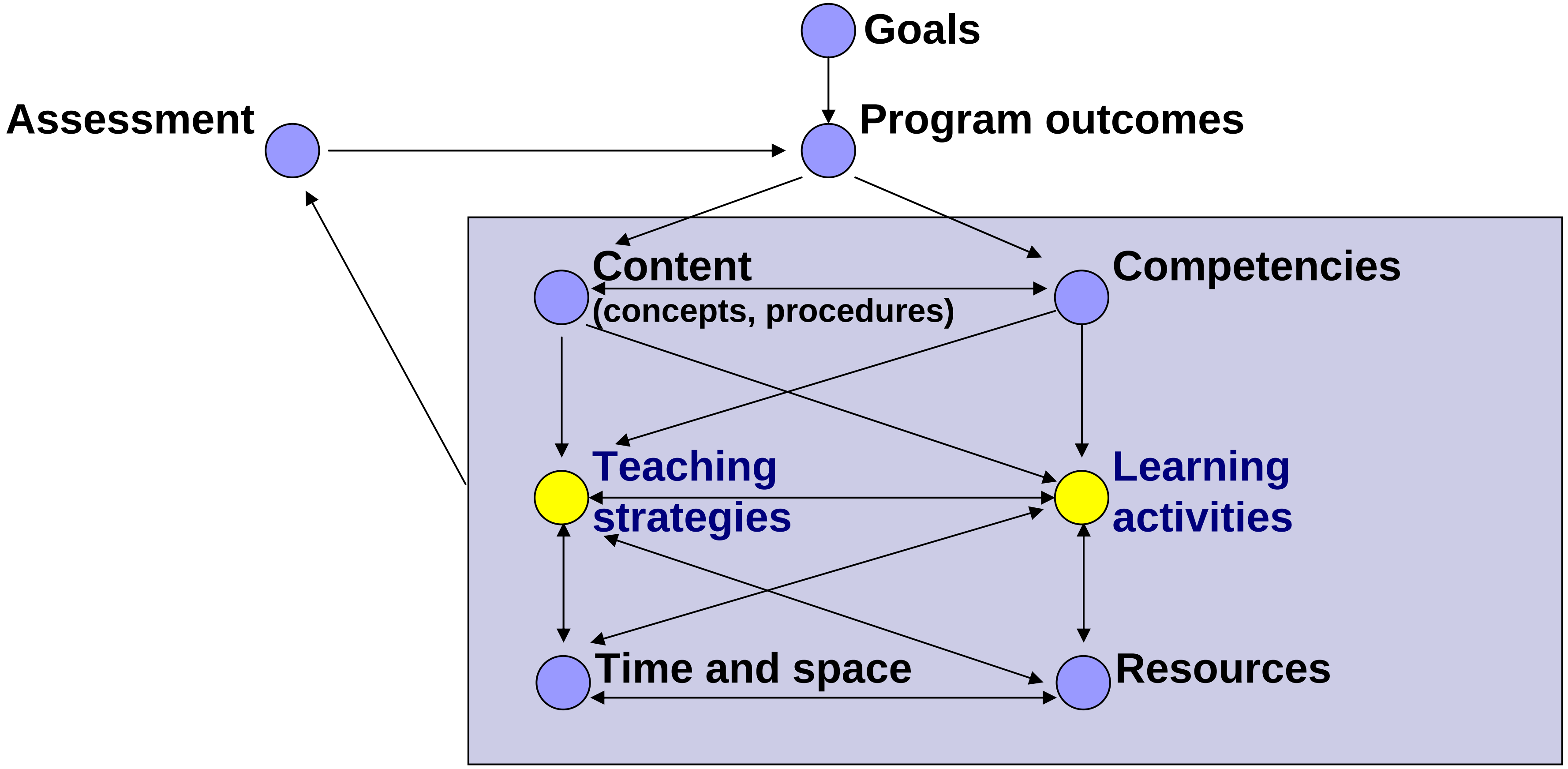


Redefining time and space for learning

- Capacity to do more… and more rapidly
  - Tackle more complex mathematical objects
  - Expand visualization and manipulation possibilities
  - Solve more realistic and complex problems
  - Handle rich and large sets of data
  - Perform various experiences, explorations, simulations
- Potential to adapt to individual learning speed
- In-class and off-class learning environment
- New window onto « the mathematical world »

Pioneer vision

Delegate technical work to the tool
and focus on conceptual work (Heid, 1988)



Typical pioneer CAS learning activities

- Use of multiple representations (symbolic, numerical, graphical) for favoring conceptual understanding
- Exploration and pattern investigation (e.g. derivatives)
- Problem solving which goes beyond what can be done in a reasonable amount of time with pen and paper
- Greater presence of *applications* (≠ *modelling*)

Obstacles and issues

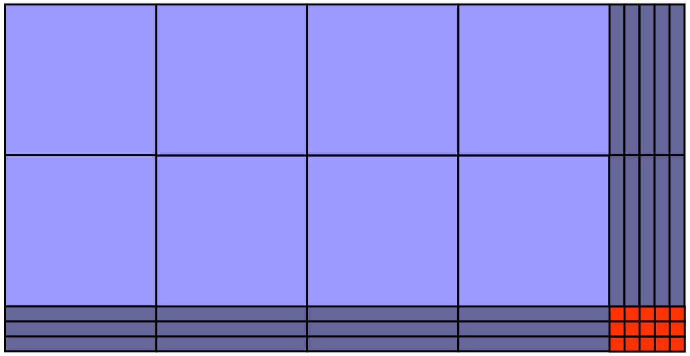
- Limits to the benefits of visualization
- “Fishing behaviour” and other “bypasses” of the task (Artigue, 1997)
- Perceived disappearance of the necessity of validating/proving
- Discrepancies between mathematical knowledge and its computational transposition in a computer (Balacheff, 1994)
- Use of black boxes (Buchberger, 1989; Drijvers, 2000)
- Impossible split between technical and conceptual (Lagrange, 2000)

A simple example
23 x 45

A black box ?

```
   23
 x 45
-----
  115
  92
-----
 1035
```

- Traditional pen & paper procedure
  - $(20+3)\times(40+5) = (5\times3) + (5\times2\times10) + (4\times10\times3) + (4\times10\times2\times10)$
    $= [(5\times3) + (5\times2)\times10] + [(4\times3)\times10 + (4\times2)\times100]$
  - Numbers as sums of powers of tens
  - Multiplication as a distributive operation
  - Addition & multiplication as commutative & associative
  - Products of powers of tens
  - Numbering system



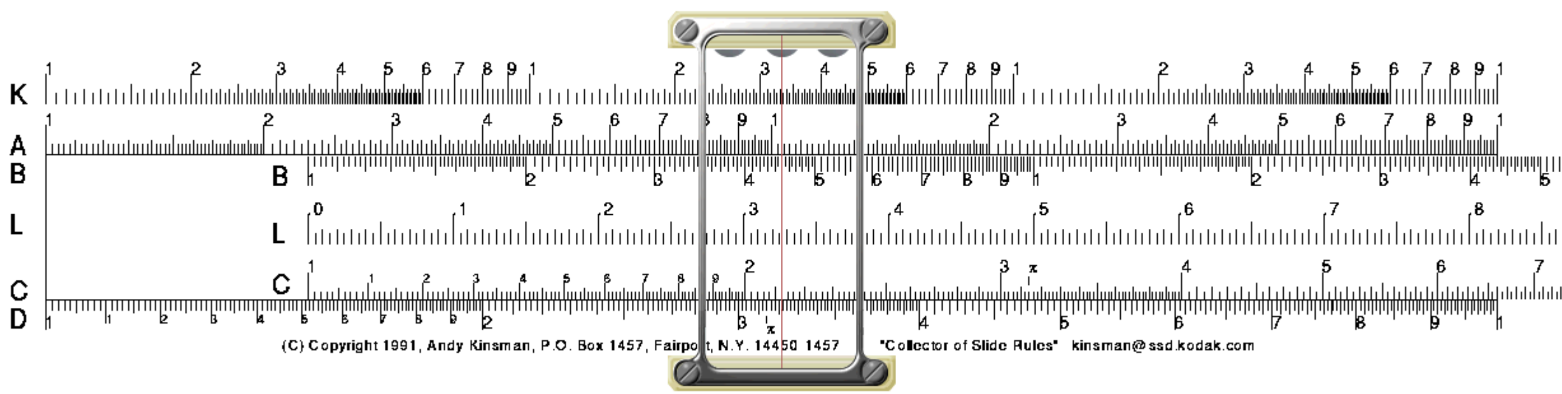
A simple example
23 x 45

- Slide rule procedure

  $(2{,}3\times10) \times (4{,}5\times10) = (2{,}3\times4{,}5) \times 10\times10 \approx 10{,}3 \text{ x } 100 = 1030$

  since $\log(2{,}3 \times 4{,}5) = \log(2{,}3) + \log(4{,}5) \approx \log(10{,}3)$

  - Numbers as products of a power of ten
  - Multiplication as associative operation on real numbers
  - Properties of logarithms
  - Interpolation, approximation and order of magnitude



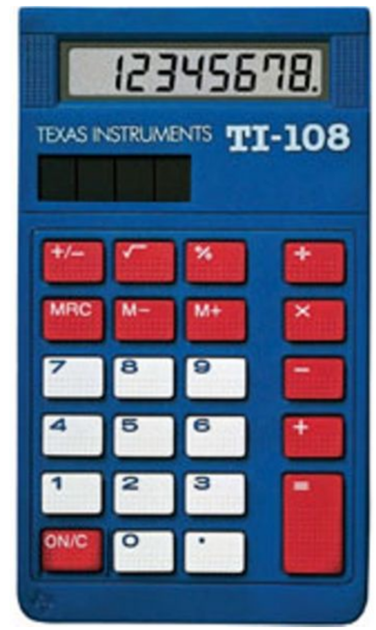
A simple example
23 x 45

- Calculator procedure

  2 3 x 4 5 =

  - ?
  - Equality?
  - Approximation and order of magnitude?
  - Properties of operations? Powers of tens?

Combine the tool and the math
to go beyond the tool's limitations.



How about computing

$23\,786 \times 45\,974$ ***?***

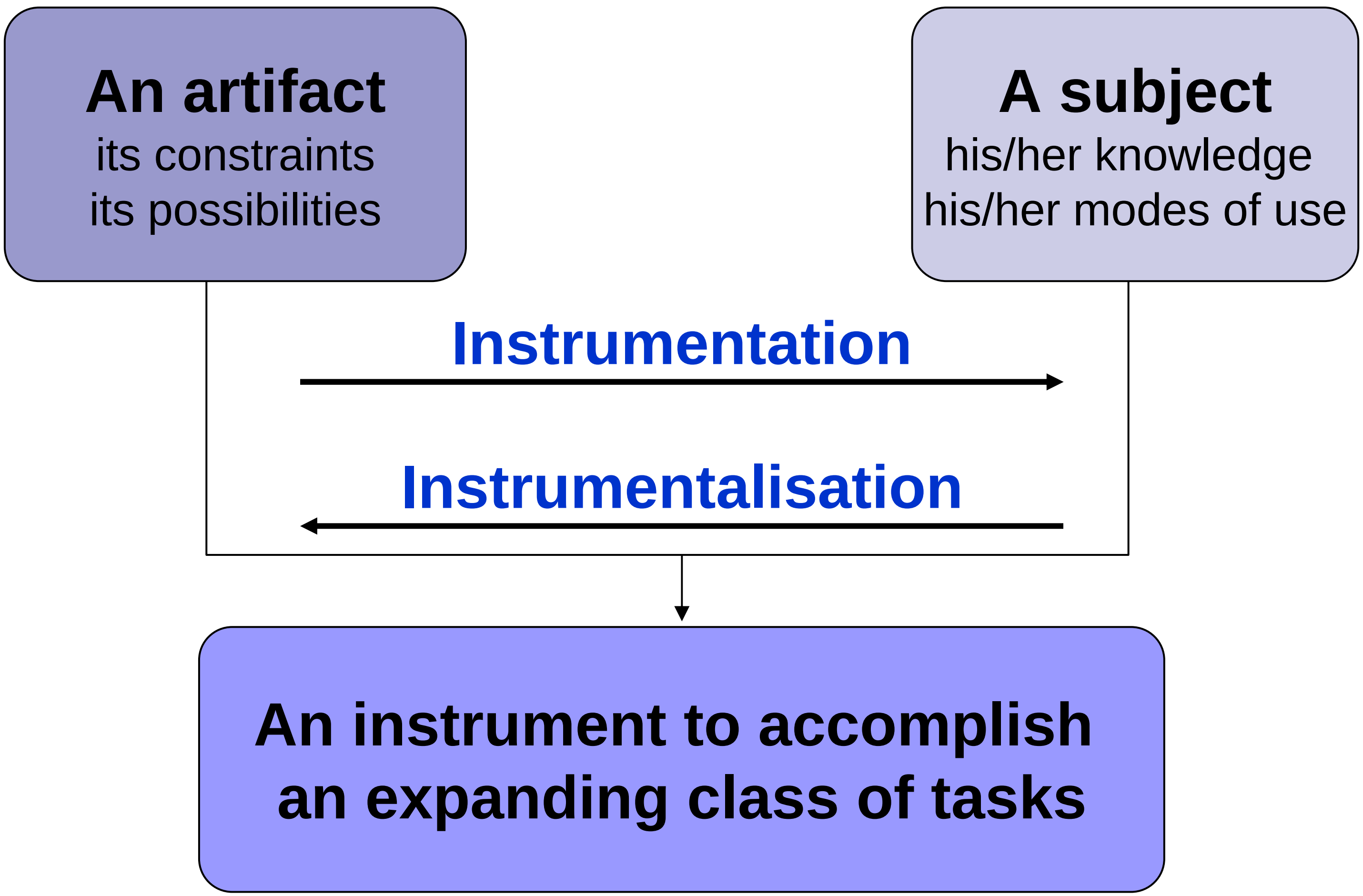
$$(23\,000 + 786) \times (45\,000 + 974) =$$

$$(23 \times 45) \times 1\,000\,000$$

$$+ (23 \times 974 + 786 \times 45) \times 1\,000$$

$$+ \; 786 \times 974$$

The instrumental approach (Rabardel, 1995)

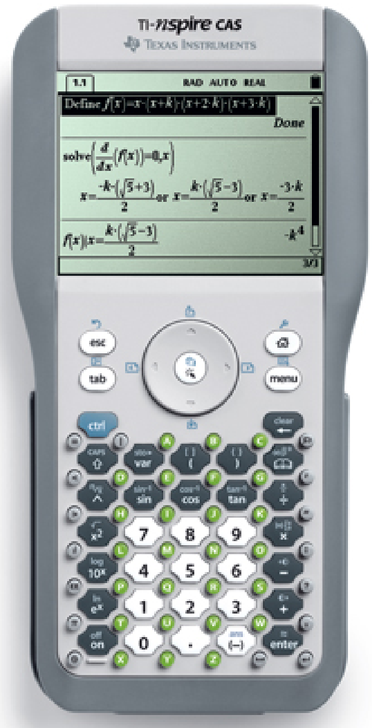


A slightly more complex example

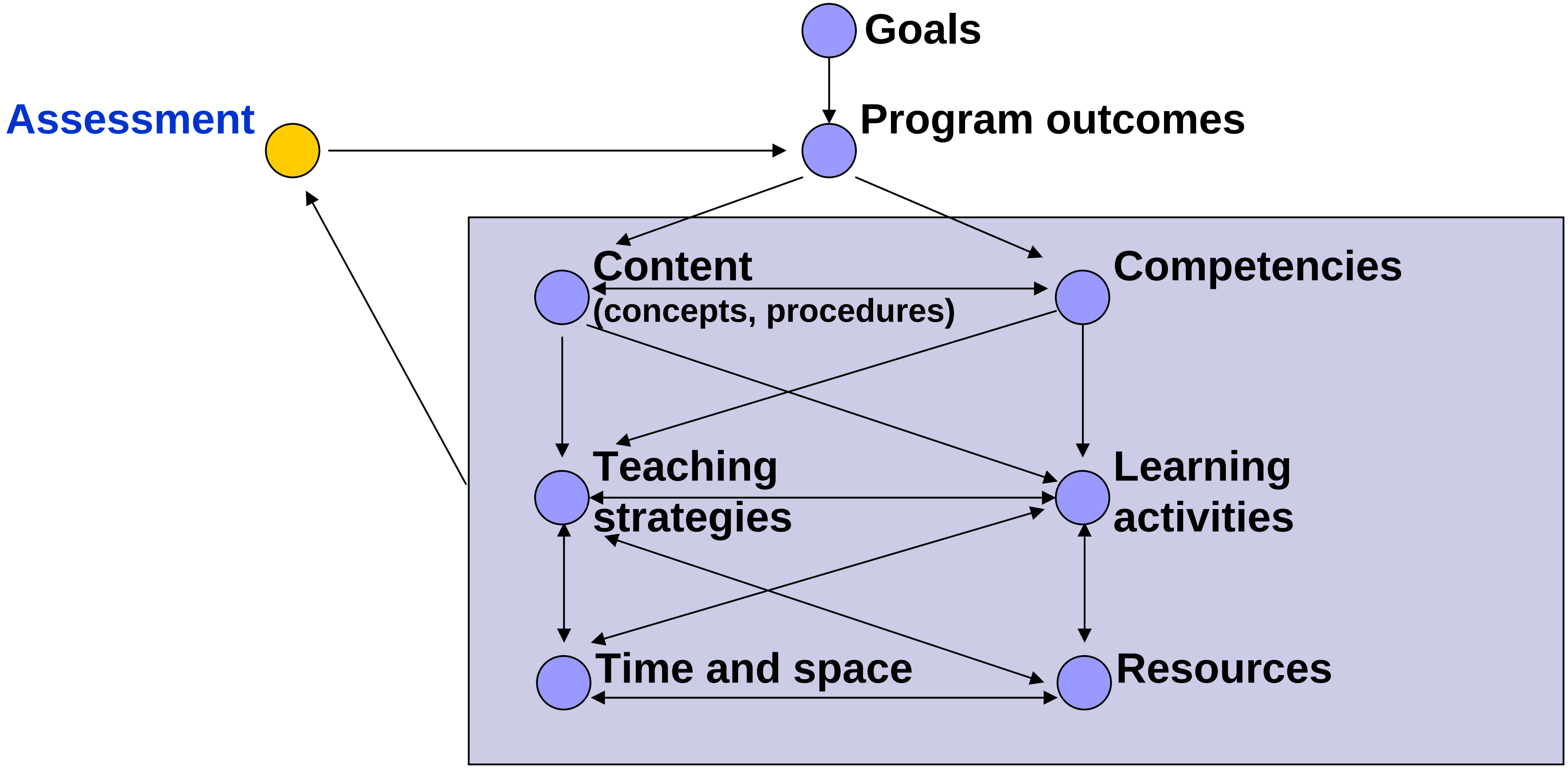
Factor $2x^3-8x^2+5x+5$

Develop alternate techniques for doing a common task.

- consolidate network of concepts
- develop autonomy and creativity with respect to the tool.



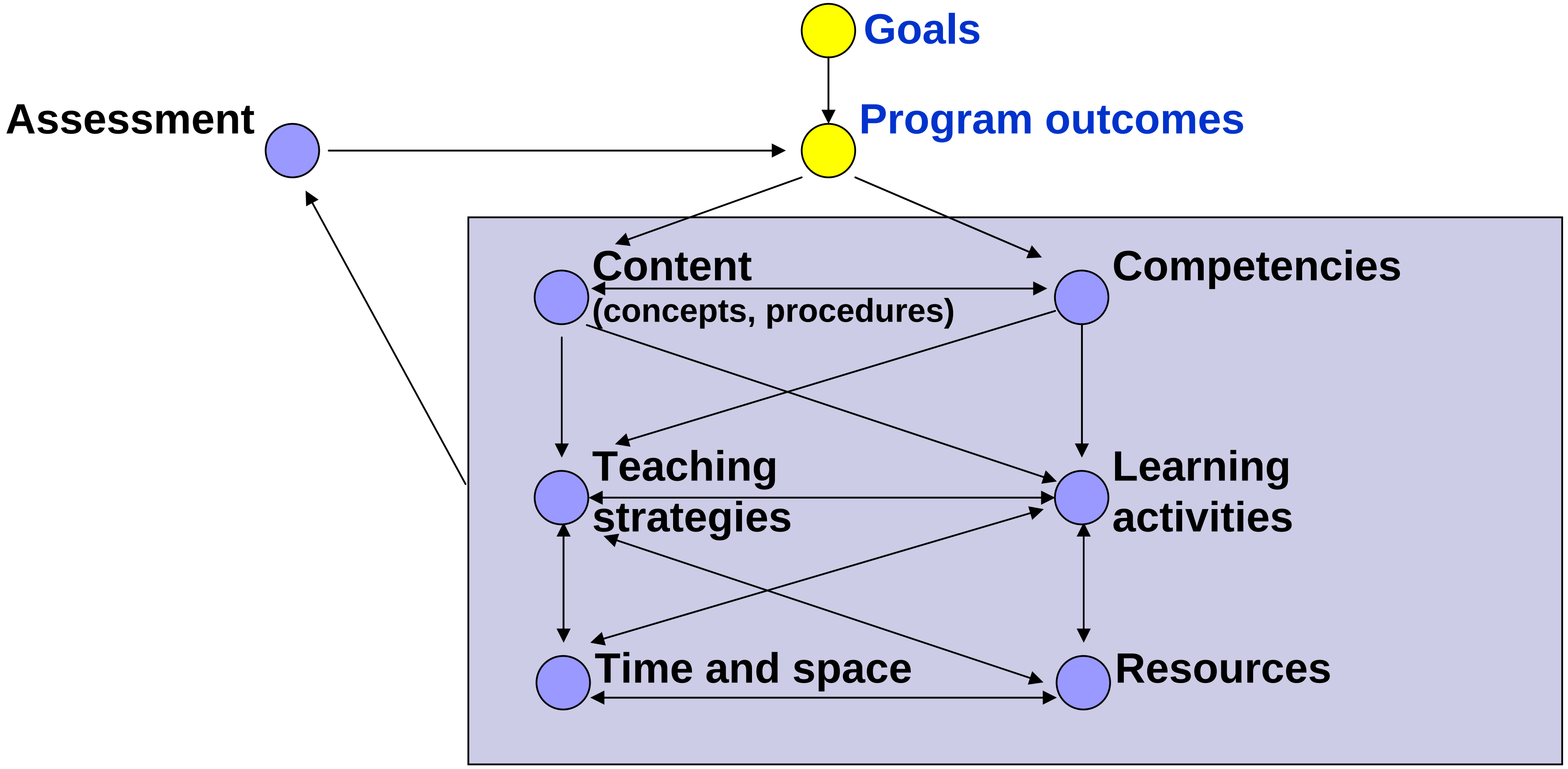
A word on assessment



A word on assessment

- Variable place and role of CAS in assessment
  - Status of the new techniques?
- Loss of information and structure in students written records
  (Cannon & Madison, 2003; Ball & Stacey, 2003)
  - Hard to assess/validate
  - Opportunity to set new norms?

Coming back to the goals

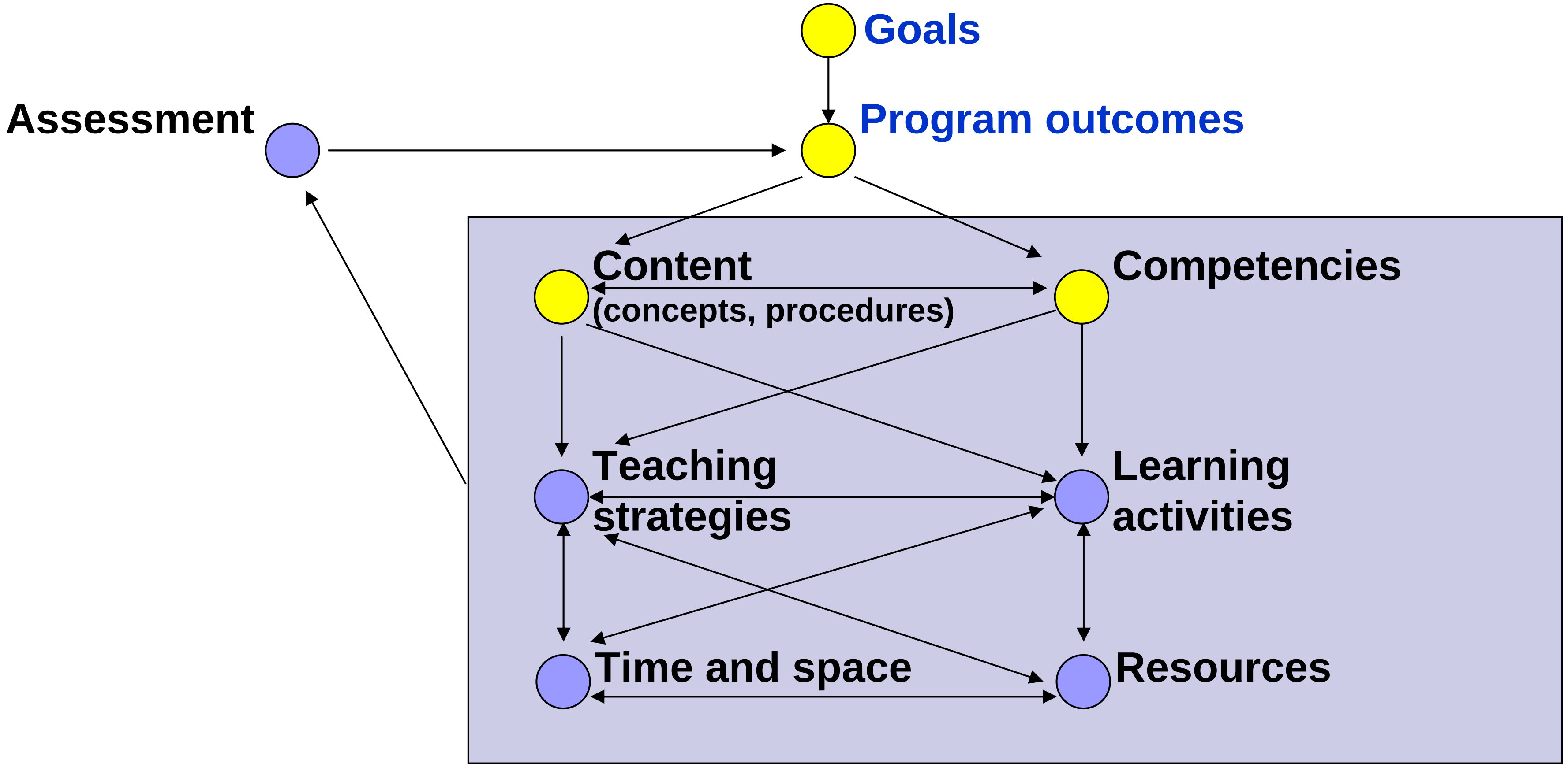


Rethinking the goals

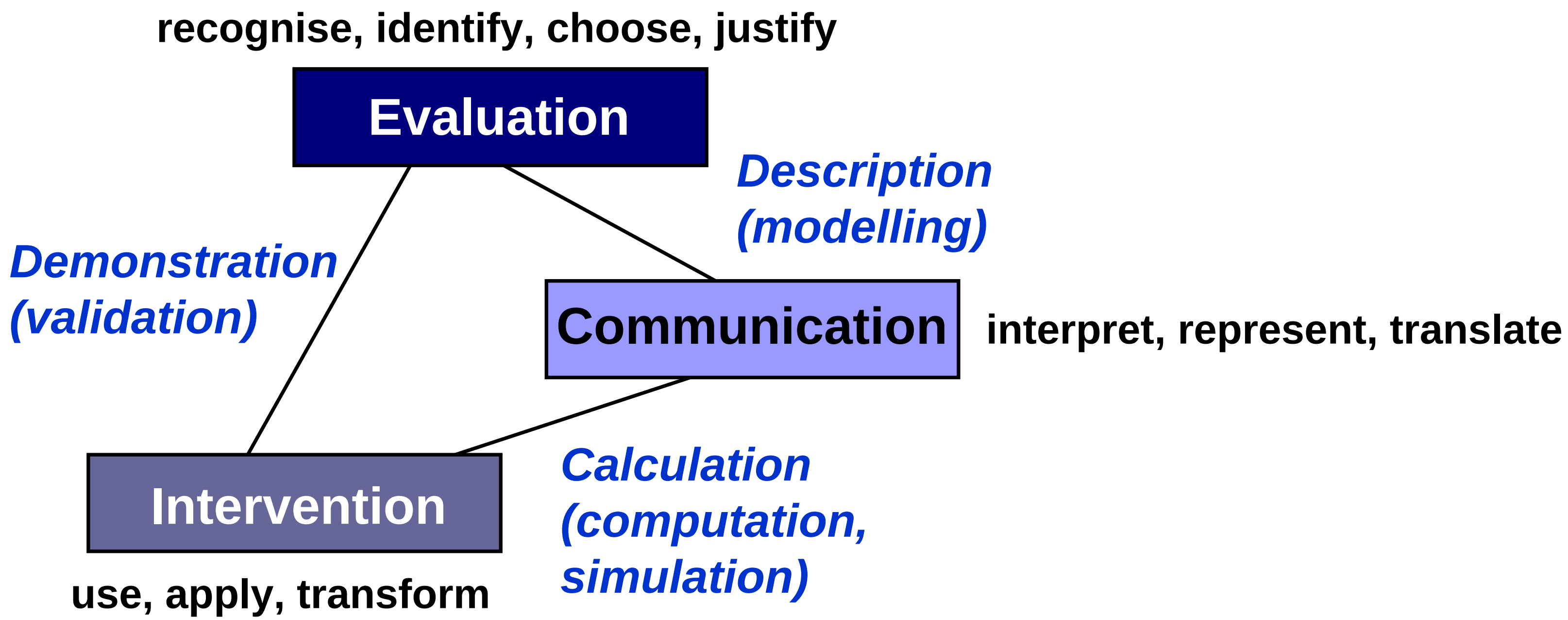
- Mathematical objects do not exist on their own, but have emerged from systems of practices with the development of techniques. (Chevallard, 1999)
- With the tremendous development of computer science, the new description possibilities introduced by mathematics translate into new capacities for action. We are entering the era of modelling. (Bouleau, 2000)
- The main purpose of technology integration in the postsecondary curriculum may well be to allow a contemporary analysis of complex systems (e.g. environment) through the power of mathematical computation and modelling.

  This might help move mathematical thinking back into the mainstream of science. (Taylor, 2008)

Rethinking contents and skills



Defining mathematical competencies



(De Terssac, 1996)

Some challenges for the future

- Better address the variety of models
  - Continuous and discrete
    - Functions and differential equations
    - Sequences and difference equations
    - Compartmental models
    - Multi-variable, …
    - Geometric models
  - Deterministic and stochastic
- Make use of « new » structures to model
  - Lists, graphs, …
- Establish dialog with experts from areas of application and other disciplines
- Value reading and writing

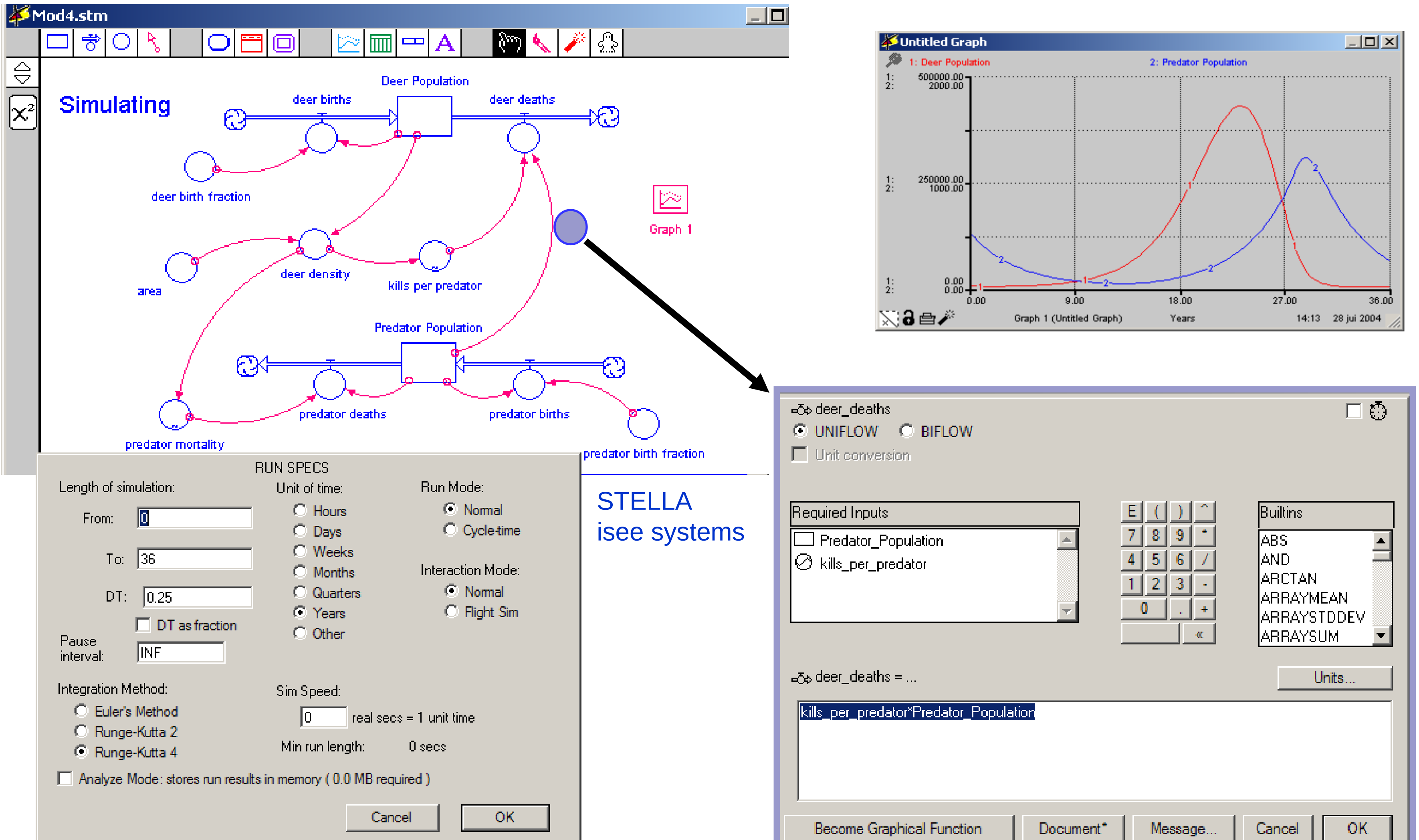
Can we move beyond RLC circuits and mass-spring systems?

Evaluation — ***Description (modelling)*** — **Communication**

Some challenges for the future

- Build bridges between methods of solving
  - Analytic, qualitative and numerical
  - Optimization and simulation
- Consider as program outcome to have students
  - be fluent at using various S/W for modelling and computing (spreadsheets, CAS, numerical S/W, DGS, …)
  - be able to perform some programming to adapt to the specifics of a problem and develop autonomy in exploring

How modelling is sometimes done in the rest of the world …

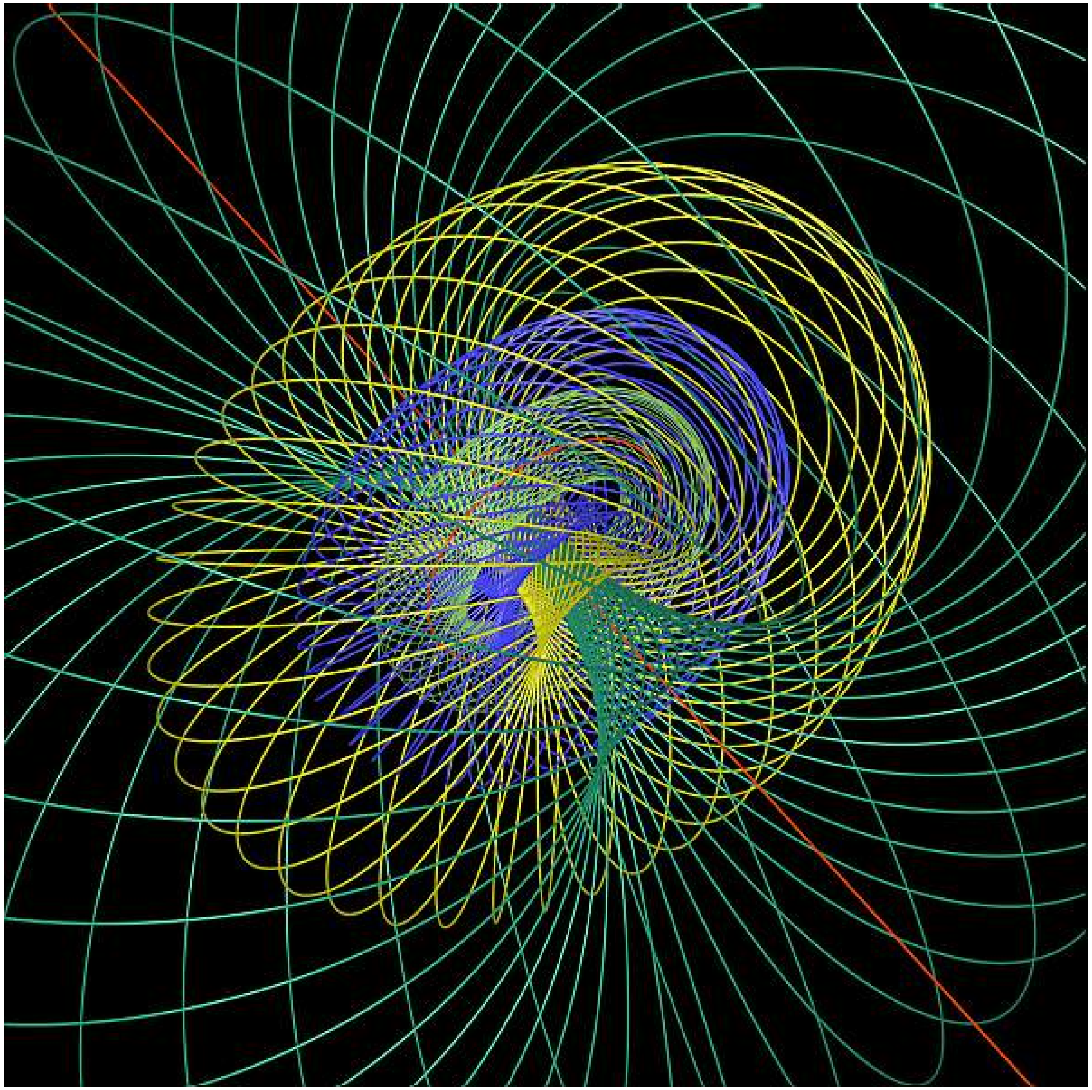


Some challenges for the future

- Develop ways of getting insight on equations that can only be solved numerically
  - From analytical and qualitative methods: transferrable key concepts at the heart of the techniques? use on meta-models?
  - From statistical methods for stochastic models
- Open as many black boxes as possible
  - Acknowledge mathematics materialisation with technology
  - Claim back math ownership of technology, or at least part of it
  - Use hard-to-open boxes as opportunities for developing modeling
- Continue to nurture imagination and sense of wonder to encourage looking for patterns, proofs and explanations

Evaluation — *Demonstration (validation)* — **Intervention**

… and a donut, please.



Limits to the benefits of visualization

« *I don't recall what it was, but I remember we would enter a series of commands and then at the end, a graph would appear. We would then change the commands, and the graph would turn or have another shape. But I don't recall what it was…* »

Helga, student at Polytechnique
describing her use of Maple
in college linear algebra

- Attention redirected towards the tool interface.
- Passive observation :

  Seeing does not automatically lead to understanding…
- Short memory retention.

Favour instrumentation process with the use of the same tool over an extended period of time.

Guide exploration with inquiry questions that lead to look for reasons behind observed behaviour.

Discrepancies between mathematics and its computational transposition

Internal constraints linked to

- Material nature of the tool
  - Finite representation of numbers in memory
  - Finite number of pixels on the screen
- Programming choices
  - Algorithms used
  - Command syntax
  - Modes of representations (Artigue, 1997)

Control the use of the graphical and numerical registers.

Look for and present to students situations where the tool fails.

Help develop ways of reconciling tool's output with expected results.

The black box issue

- Conflict between pragmatic and epistemic considerations
- Knowledge required to open some of these boxes beyond current math curriculum or the school level at which they are first introduced
  - Particularly true with CAS (Artigue, 2002).
- Consequent
  - loss of control over tool's output
  - feeling of discomfort among many teachers/professors
  - potential for students' investigation
  - sense of frustration among some students (Drijvers, 2000)

Accept...

… or uncover, whenever possible.

machine only if you examine it afterwards personally and if you see, by actual use, how it is operated. The machine will be at your disposal, for that purpose, after the lecture.

Permit me to summarize by remarking that *the theoretical principle of the machine is quite elementary and represents merely a technical realization of the rules which one always uses in numerical calculation.*

I think that the arrangement of the machine will

describe to you the process of carrying out a definite

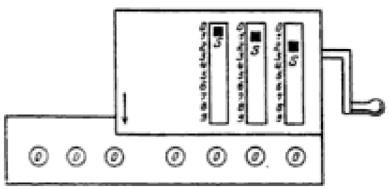


Fig. 1. Before the first turn.

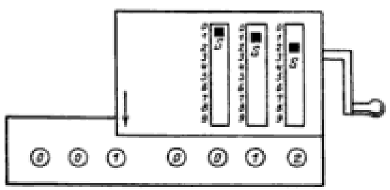


Fig. 2. After the first turn.

the way in whic
brings it about. I
multiplication.
The procedur
One *first sets the*
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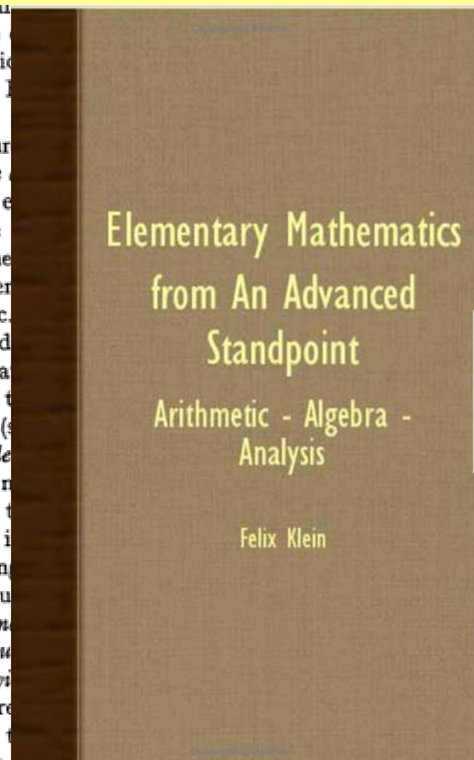
lever at the one
cond at the ten
multiplicand, etc.
the multiplicand
the first lever a
lever at 1; all t
remain at zero (
turn the handle
clockwise. The n

pears under the openings of the slide, in our case a 2 in t
from the right, a 1 in the second, while zeros remain i
Simultaneously, however, in the first of a series of openin
the left, the digit 1 appears to indicate that we have tu
once (Fig. 2). *If now one has to do with a multiplier of on*
the handle as many times as this digit indicates; the mu
be exhibited on the slide to the left, while the product wi
slide to the right. How does the apparatus bring this re
the first place there is attached to the under side of t
left, a cogwheel which carries, equally spaced on its
0, 1, 2, . . ., 9. By means of a driver, this cogwheel is rotated through one tenth of its perimeter with every turn of the handle, so that a digit becomes visible through the opening in the slide, which actually indicates



the number of revolutions, in other words the multiplier. Now as to the *obtaining of the product*, it is brought about by similar cogwheels, one under each opening at the right of the slide. But how is it that
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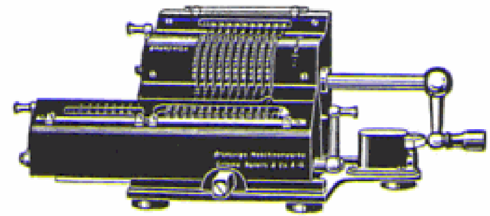

Fig. 3.

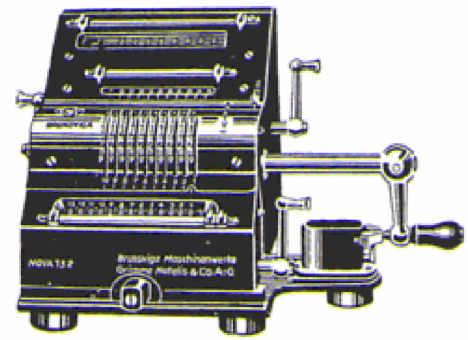

Fig. 3a.

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. These teeth
under the corre-
of the slide, so
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S. Accordingly,

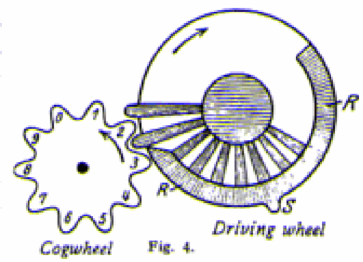

Fig. 4.

ration, when we start at the zero position, and
nce, the units wheel must jump to 2, the ten's
12 appears. A second turn of the handle moves
other 2 and the tens wheel another 1, so that 24 appears, and similarly, we get, after 3 or 4 times, $3 \cdot 12 = 36$ or $4 \cdot 12 = 48$, respectively.