



Forensics, Mathematics And A CAS II



Carl Leinbach
Professor Emeritus
Gettysburg College
leinbach@gettysburg.edu

Patricia Leinbach
Adams County Coroner, retired
accoroner@embarqmail.com

Some Topics Mentioned in Part I

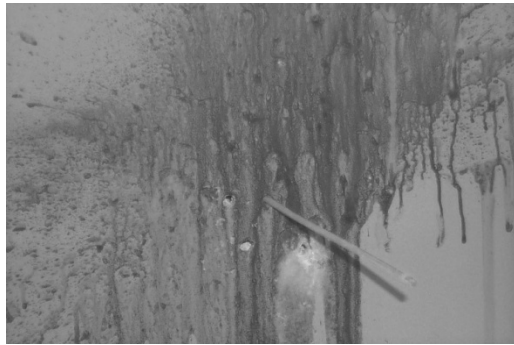


Height from Stride – Linear Regression

blood stain	Bob	Sue	John	Lisa
████████	████████	████████	████████	████████
████████	████████	████████	████████	████████
████████	████████	████████	████████	████████
████████	████████	████████	████████	████████
████████	████████	████████	████████	████████
████████	████████	████████	████████	████████
████████	████████	████████	████████	████████
████████	████████	████████	████████	████████



DNA and Fingerprint Evidence – Probability & Statistics



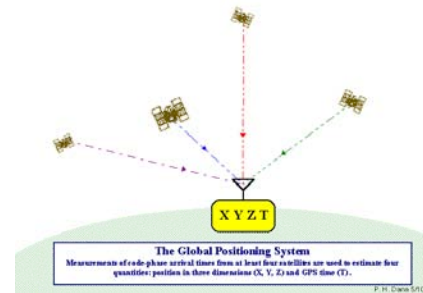
Blood Spatters – Geometry & Trigonometry



Time Since Death – Exponential Regression and Numerical Approximation

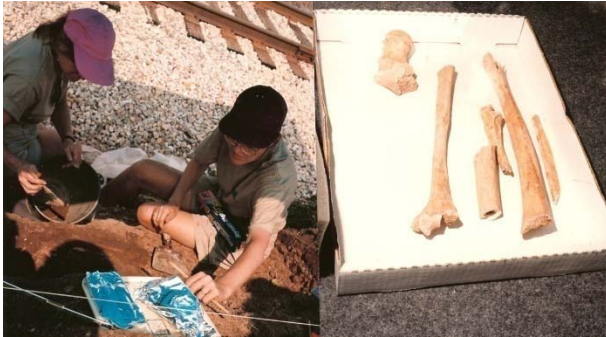


Accident Reconstruction – Equation Solving



GPS – Geometry and Systems of Equations

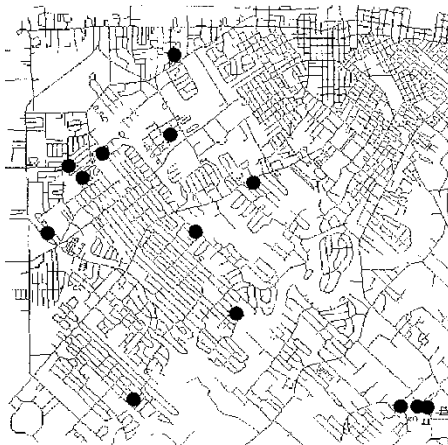
Other Related Topics of Interest



Identification of Physical Characteristics from Skeletal Evidence – Logic and Statistics



Forensic Decision Making – Analytical Hierarchical Ranking Process and Matrix Multiplication



Searching for a Serial Killers Home Base – Ranking, Probability, & Decision Making

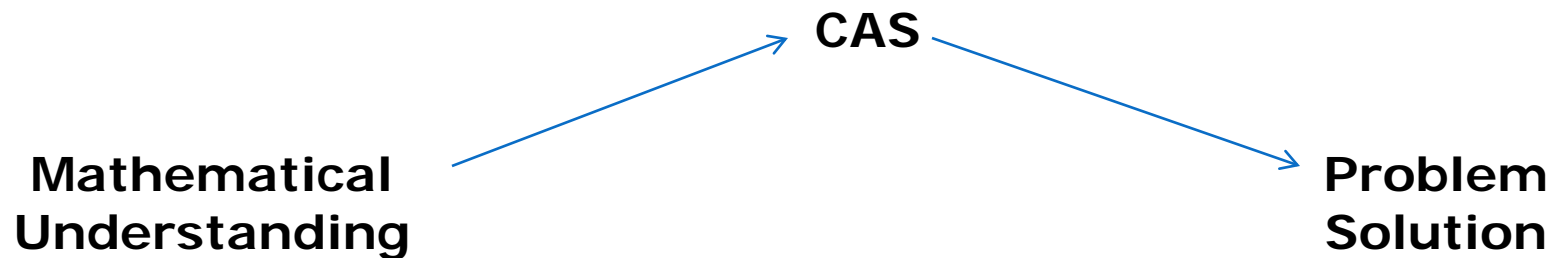
Organizing an In Class Forensic Investigation

- Teams are best
 1. Ideal size 3-4 students
 2. Have a planning session at the “crime scene” to determine what evidence should be collected to take back to the “lab”.
 3. Instructor is strictly a “guide”, let students decide how to proceed.
- Allow all ideas to flow
 1. Do not let the mathematics dictate the method of analysis.
 2. Be open to different approaches.
 3. Remember, when you don't know what to do, you know what to do.
 - a. Identify variables
 - b. Identify relationships
 - c. Graph
 4. Do not be adverse to *empirical* methods, i.e. line of site as first attempt at a linear regression.
 5. Gently suggest improvements or alternatives.
- Make sure concepts are understood by student investigators
 1. Stress the mathematical idea behind the analytical method
 2. Explain why the method makes sense within the particular context
 3. Try out the method with a simple example that involves the application of the concept.
- Involve the use of the CAS.

What is the Role of the CAS?



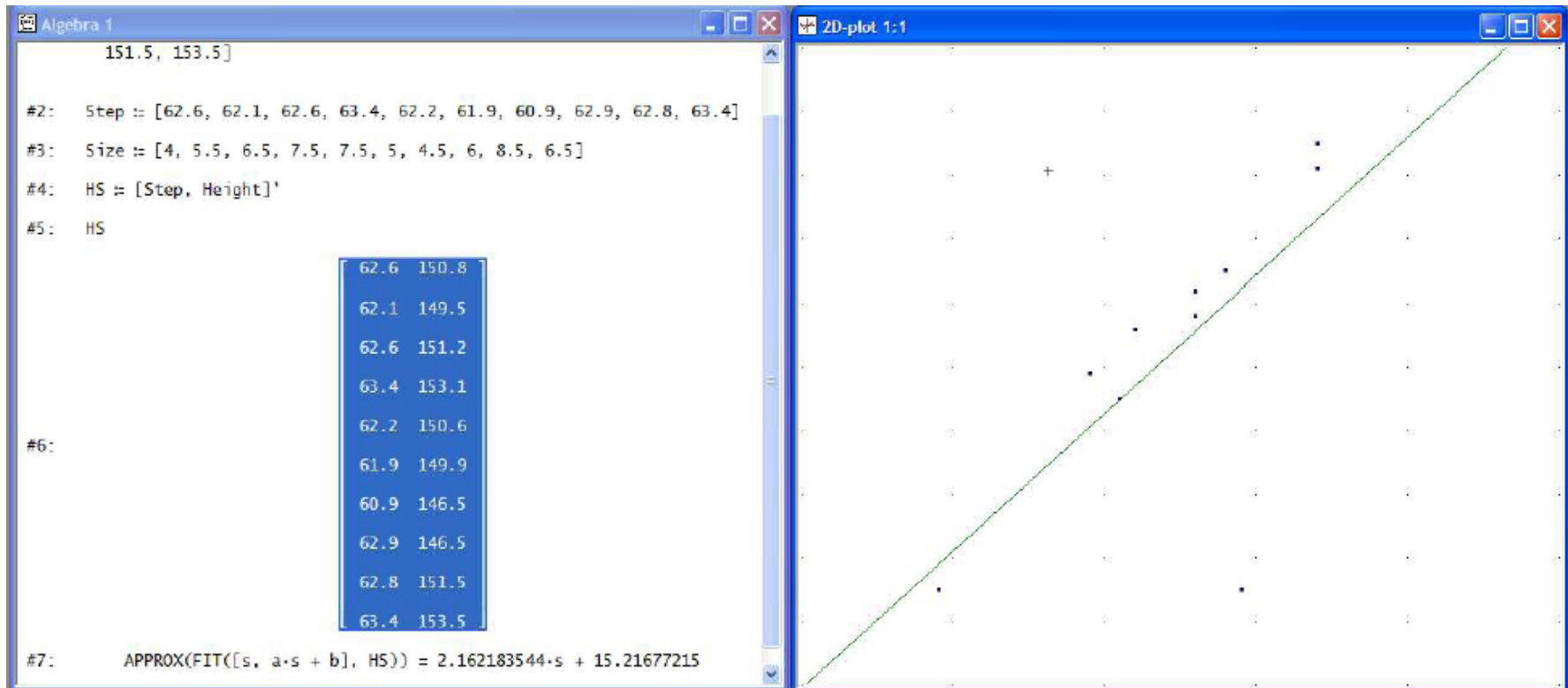
It is a bridge



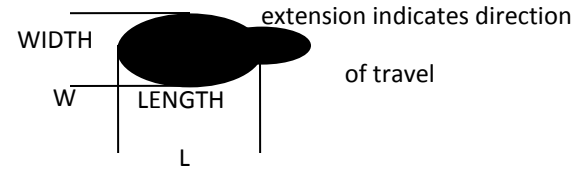
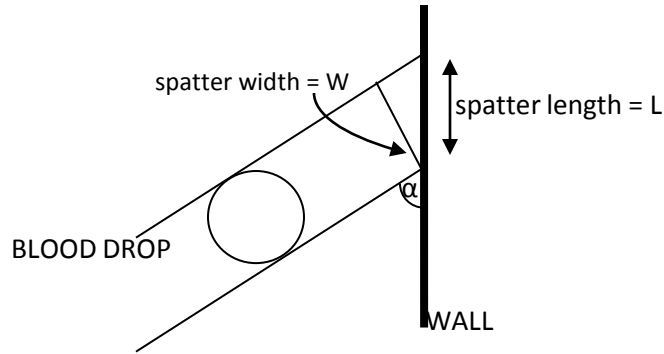
Mathematical Understanding must be the shibboleth for crossing the bridge.
Otherwise, we have an exercise in "button pushing."
CAS is a tool, not the solution.

Some Examples of Solving Forensic Problems

Estimating Height From Stride Length



Blood Spatters



$$\sin(\alpha) = \frac{W}{L}$$

The Physics of the Formation of a Blood Spatter

Drawing a "blood spatter" in the CAS with $\alpha = 1$ radian

Algebra 1

2D-plot 1:1

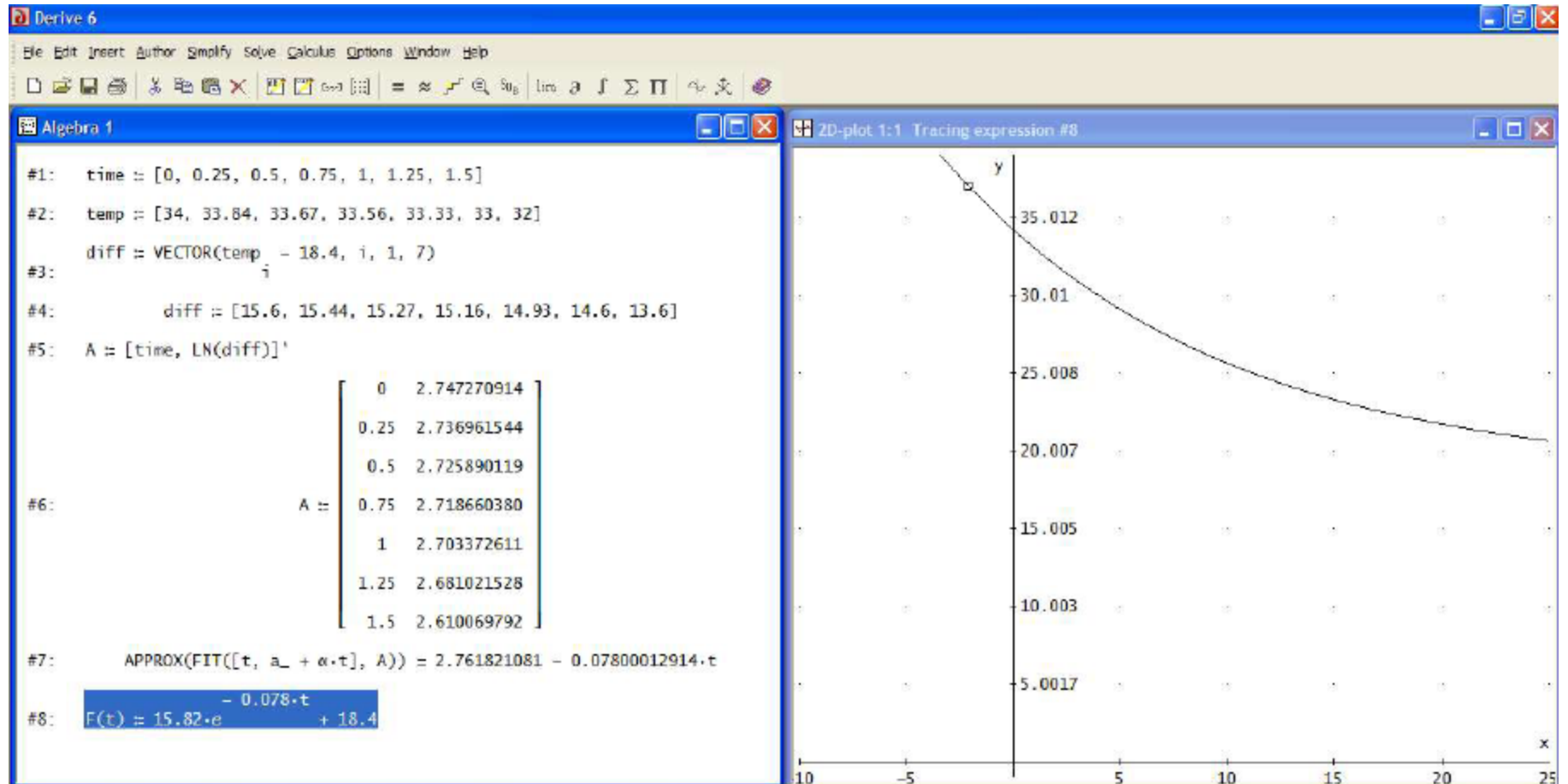
#1: $[3 \cdot \cos(t) + 5, \sin(t) + 7]$

#2: $\begin{bmatrix} \cos(1) & \sin(1) \\ -\sin(1) & \cos(1) \end{bmatrix}$

#3: $[3 \cdot \cos(t) + 5, \sin(t) + 7] \cdot \begin{bmatrix} \cos(1) & \sin(1) \\ -\sin(1) & \cos(1) \end{bmatrix}$

#4: $[1.620906917 \cdot \cos(t) - 0.8414709848 \cdot \sin(t) - 3.188785364,$
 $2.524412954 \cdot \cos(t) + 0.5403023058 \cdot \sin(t) + 7.989471065]$

Estimating Time Since Death – Constant Ambient Temperature



Estimating Time Since Death – Variable Ambient Temperature

Derive 6 - [Algebra 1]

File Edit Insert Author Simplify Solve Calculus Options Window Help

#9: $tim := [-5, -4.5, -4, -3.5, -3, -2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5]$
 #10: $amb := [22.2, 21.7, 21.1, 20.4, 20, 19.3, 19.2, 18.9, 18.9, 18.5, 18.5, 18.4, 18.4, 18.3]$

```

K(t, tim, amb, i, n) :=
  Prog
  n := DTFNSTON(tim)
  i := 1
  Loop
  If t = timi
  #11:   RETURN ambi
  If t > timi ∧ t ≤ timi(i + 1)
  RETURN ambi + (ambi(i + 1) - ambi) / (timi(i + 1) - timi) · (t - timi)
  i := i + 1
  If i > n - 1
  RETURN 0
  
```

#12: $K(-4.2)$

#13: 21.34

#14: $VECTOR(K(-5 + 0.2 \cdot i), i, 0, 20)$

#15: $[22.2, 22, 21.8, 21.58, 21.34, 21.1, 20.82, 20.54, 20.32, 20.16, 20, 19.72, 19.44, 19.28, 19.24, 19.2, 19.08, 18.96, 18.9, 18.9, 18.9]$

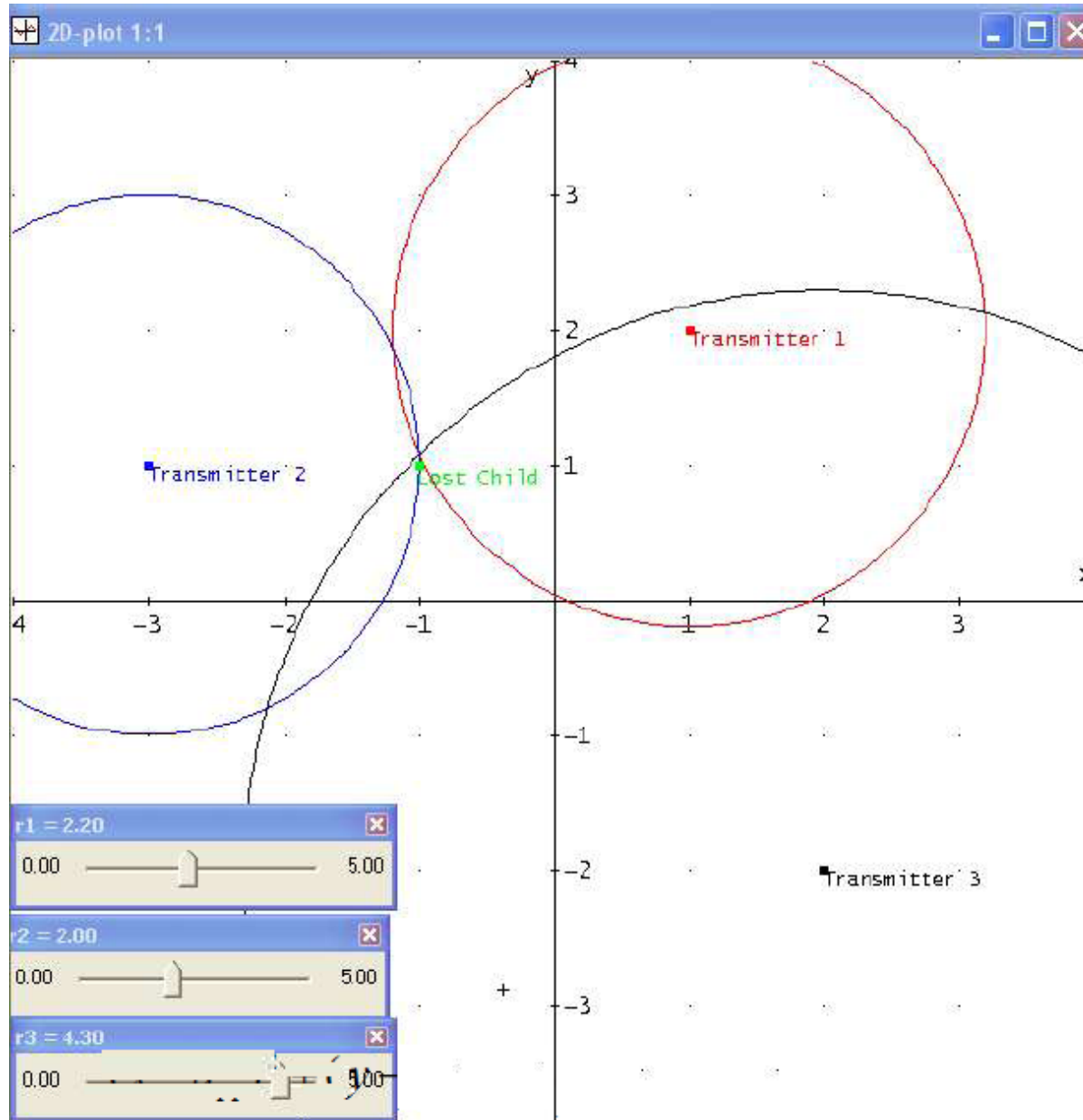
Press F1 for Help

Approx(#14) 0.040s

vector(K(-5+0.2*i), i, 0, 20)

$\alpha \beta \gamma \delta \epsilon \zeta \eta \theta \iota \kappa \lambda \mu \nu \xi \omicron \pi \rho \sigma \tau \upsilon \phi \chi \psi \omega$
 $\Lambda \text{B} \Gamma \Delta \text{E} \text{Z} \text{H} \Theta \text{I} \text{K} \text{L} \text{M} \text{N} \Xi \text{O} \text{P} \text{R} \Sigma \text{T} \Upsilon \Phi \text{X} \Psi \Omega$
 $[\{ + \cdot ^ \% = < \leq \sqrt{-} \backslash \cup ' := e \pi \infty \partial \Sigma \Gamma \zeta \chi$
 $] \} _ / \sqrt{_} \perp \# > \geq \wedge \cdot \equiv \cap \downarrow \ll \dot{_} \gamma ^\circ \int \Pi \psi \times \cdot$

An Approximate Graphical Solution to the 2D GPS Problem



An Analytic Solution to the 2D GPS Problem Using A System of Nonlinear Equations

Derive 6 - [Algebra 1 Lost Child.dfw]

File Edit Insert Author Simplify Solve Calculus Options Window Help

The speed of light in km/μsec

#1: $c := 0.299792458$

The time stamp on the receiving unit

#2: $t := [6.377871991, 6.299128190, 7.047192602]$

The three equations to be solved to locate the position. All positions are given in kilometers

#3: $(x - 0.1)^2 + (y - 0.2)^2 = c^2 \cdot (t_1 - \delta)^2$

#4: $(x + 0.3)^2 + (y - 0.1)^2 = c^2 \cdot (t_2 - \delta)^2$

#5: $(x - 0.2)^2 + (y + 0.2)^2 = c^2 \cdot (t_3 - \delta)^2$

Using the Derive 6.1 solve command

#6: $\text{SOLVE}\left[\left[(x - 0.1)^2 + (y - 0.2)^2 = c^2 \cdot (t_1 - \delta)^2, (x + 0.3)^2 + (y - 0.1)^2 = c^2 \cdot (t_2 - \delta)^2, (x - 0.2)^2 + (y + 0.2)^2 = c^2 \cdot (t_3 - \delta)^2\right], [x, y, \delta]\right)$

Note that there are two possible solutions to this set of equations. This is a result that there could be two values for δ . We note that the second is the solution we found using the geometric method. Recall the position is given in kilometers, δ in μseconds.

#7: $[x = 0.0072322271700 \wedge y = -0.1830929452 \wedge \delta = 7.692664945, x = -0.09999999999 \wedge y = 0.09999999999 \wedge \delta = 5.631999999]$

Simp(#6) 0.031s

Conclusions

1. Many teenagers are interested in Forensics. Television fuels this interest.
2. Mathematics plays an important role in answering questions that arise in Forensic Investigations.
3. The basic mathematical concepts are easily understood and accessible to teenagers. Applying the techniques may be tedious and/or just beyond their reach.
4. The CAS can bridge the gap and hopefully evoke a "How'd they do that?" response within some students.
5. The CAS will never replace reasoning, but can definitely assist it.

Contact Us

Pat: accoroner@embarqmail.com

Carl: leinbach@gettysburg.edu