VECTOR SPACE BASES ASSOCIATED TO VANISHING IDEALS OF POINTS

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Let $\mathbb{k}[x_1, \ldots, x_n]$ be the polynomial ring in n variables over a field $\mathbb{k}$. The vanishing ideal with respect to a set of points $\{p_1, \ldots, p_m\}$ in $\mathbb{k}^n$ is defined as the set of elements in $\mathbb{k}[x_1, \ldots, x_n]$ that are zero on all of the p_i's.

The main tool that is used to compute vanishing ideals of points is the Buchberger-Möller algorithm, described in [1]. The Buchberger-Möller algorithm returns a Gröbner basis for the ideal vanishing on the set $\{p_1, \ldots, p_m\}$. A complementary result of the algorithm is a vector space basis for the quotient ring $\mathbb{k}[x_1, \ldots, x_n]/I$. However, in many applications it turns out that it is the vector space basis, rather than the Gröbner basis of the ideal, which is of interest. For instance, it may be preferable to compute normal forms using vector space methods instead of Gröbner basis techniques.

A new bound for the arithmetic complexity of the Buchberger-Möller algorithm is given in [3], and is equal to $O(nm^2 + \min(m, n)m^3)$. We will discuss four constructions of vector space bases, all of which perform better than the Buchberger-Möller algorithm. An application of the constructions will be that we can drastically improve the method of the reverse engineering of gene regulatory networks given in [2].

The paper behind this work is accepted for publication in Journal of Pure and Applied Algebra. A preprint is availible at http://arxiv.org/pdf/0808.3591/v2.

References

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